

Neutron Stars

- **Goals:**
 - What becomes of stars after they undergo a supernova
 - How do we detect neutron stars
 - How are they laboratories for General Relativity
- **After the Supernova**
 - **The collapse of the core**
 - The pressure generated by the collapse forces most of the e^- and p^+ to combine into neutrons.
 - This forms a degenerate gas (as with white dwarfs) that can support the neutron star.
 - Like the Chandrasekhar limit there is a mass limit on the neutron star ($\sim 3M_\odot$).
 - End with a rapidly rotating core of neutrons with a density $4 \times 10^{17} \text{ kg m}^{-3}$ (the Earth would be 250 m across).
 - Size of the collapsed core is about 20 km.

- **Conservation of angular momentum**

- **Angular momentum**

- Angular momentum is defined as,

$$\mathbf{L} = \sum \mathbf{m}_i \mathbf{v}_i \mathbf{r}_i$$

- **L**: angular momentum
m_i: mass of part i of an object
r_i: radius of mass i
v_i: velocity of mass i

- **Conservation**

- Angular momentum is conserved in the same way that momentum is conserved (Newton's first law).
- As the radius of an object (star) decreases the velocity must increase to conserve L.
- Assume the Sun ($R=7 \times 10^5$ km) collapsed to the size of a neutron star ($R=20$ km).
 Velocity increases by 35000x.
- Velocity at surface of sun (v_{surface})

$$V_{\text{surface}} = \frac{2\pi r}{P}$$

- **P**: period of rotation (3×10^6 s)
R: radius of Sun (7×10^5 km)
- $V_{\text{surface}}(\text{Neutron star}) = 51 \times 10^6 \text{ m s}^{-1}$.
- $P(\text{Neutron star}) = 2.4 \times 10^{-3} \text{ s}$

• Density of Neutron Stars

– Rotational velocity

- For a star to remain intact the centripetal acceleration < gravitational acceleration.

$$\frac{V^2}{R} = \frac{GM}{R^2}$$

V: velocity of the star at the equator

R: radius of the star

M: mass of the star

- Maximal velocity is then

$$V = \sqrt{\frac{GM}{R}}$$

– Period of a neutron star

- For a period (P) of 2ms what is the density of the star

$$P = \frac{2\pi R}{V}$$

- Period is related to mass by

$$P = \frac{2\pi R^{3/2}}{(GM)^{1/2}}$$

- Substituting for density

$$P = \frac{3.8 \times 10^5}{\rho^{1/2}}$$

- For P=0.002s: $\rho = 4 \times 10^{16} \text{ kg m}^{-3}$

- **Observing Neutron Stars**

- **Discovery 1967 (Hewish and Bell)**

- Figure 23-1**

- First radio surveys discovered variable radio stars (period of 1.337s)
 - Period too rapid for normal variable stars or eclipsing binaries (requires orbits of < 1000 km).
 - Named pulsars due to regular pulsations.
 - Predicted in 1930s by Zwicky and Baade.

- **Rotating Neutron Stars**

- Figure 23-3**

- Pulsars are rapidly rotating (milliseconds to seconds) and extremely compact and dense.
 - The magnetic fields present in the Sun are conserved (increasing in strength by a factor of 10^{10}).
 - Field $\sim 10^{12}$ Gauss (bar magnet 100 Gauss).
 - Model that describes the neutron star is the rotating lighthouse.
 - Magnetic pole and pole of rotation are not coincident. Magnetic pole rotates.
 - Neutron star acts like a giant generator creating strong electric fields.

– Rotating Neutron star

Figure 23-4

- Electric field pulls charged particles off the Fe crust of the neutron star.
- These are accelerated by the magnetic field and emitted in a tight beam.
- Accelerated electrons emit synchrotron radiation.
- If the beam intercepts with our line of sight we see a pulse of radiation (hence pulsars).
- Time between pulses is the rotation period (like a lighthouse).
- The emission of the electrons removes energy and angular momentum (the pulsar slowly slows down).
- Synchrotron radiation is emitted in the radio - we also see pulses in the X-ray and optical.
- The most well known pulsar is in the Crab nebula (period of 0.033s).
- Believed to form from Type II supernovae.
- Total energy output of the Crab Nebula is 3×10^{31} W.

– Spin down of Pulsars

- Crab pulsar outputs 3×10^{31} W of energy into the surrounding nebula.
- This energy loss slows the pulsar (period of typical pulsar increases 3×10^{-8} s yr⁻¹).
- Slow down can be used to estimate the age of a pulsar.

$$t \approx \frac{P}{dP/dt}$$

- t: age of pulsar
P: period of pulsar
dP/dt: rate of change of period
- For the crab nebula its spin down rate is about 1.2×10^{-13} s per s

$$\begin{aligned} t &\approx \frac{0.033}{1.2 \times 10^{-13}} \\ &\approx 10^{11} \text{ s} \quad (10^4 \text{ yrs}) \end{aligned}$$

- Old pulsars spin slower than young pulsars.
- Neutron stars slow down faster when they are young.

- **Interior of a Neutron Star**
 - **Superfluidity and superconductivity**

Figure 23-10

- **Core of the neutron star contains neutrons, protons and electrons.**
- **Surrounded by a crust of metals (e.g. iron).**
- **Protons and electrons anchor the magnetic field of the pulsar.**
- **Neutrons, protons move without friction (superfluidity) of electrical resistance (superconductivity).**
- **Core rotates freely as the crust slows down. The faster rotating core can deliver sharp jolts (glitches) to the crust speeding up the neutron star.**

- **Fastest Pulsars and Mass Transfer**

- **Millisecond pulsars**

- In 1982 pulsars with 1ms periods were discovered (PSR 1937+21).
- Pulsar should be extremely young and slowing rapidly but its period increases 10^{-6} x slower than the Crab pulsar.
- Fastest pulsars are usually in close in binary systems.
- If a pulsar is formed in a binary system and the second star becomes a red giant its atmosphere expands and fills its Roche lobe. Mass can then be transferred to the neutron star.
- Adding mass adds angular momentum and the neutron star gets spun up.
- Adding H and He to a neutron can lead to further rounds of nuclear burning and releases of $>10^{37}$ J of energy.

- **Mass Transfer**
 - **Equipotential Surfaces**

Figure 21-16

- Gravitational potential energy declines as $1/r$ from the center of a star.
- We can draw lines of constant potential energy (equipotential) around a star (circles orbits around an isolated star).
- If we consider 2 stars in a binary system we can draw the combined equipotentials. One of these potentials forms the shape of an hour glass shapes (∞).
- For this potential (Roche lobe) the effective gravity of the two stars is zero (they cancel out).
- The interior of the Roche lobe describes the region in which gas is gravitationally bound to a particular star.
- The Lagrangian point is the point where the lines of equipotential touch (equal pull from either star). Mass can be easily transferred from one star to the next across this boundary point.

– Pulsating X-ray sources and Novae

- The transfer of mass from a companion gets funneled to the magnetic poles.
- When it impacts the neutron star it is travelling at 0.5 speed of light and the energy released heats the polar regions to 10^8 K (X-rays released).
- Mass accretion onto the pulsar is about $10^{-9} M_{\odot} \text{ yr}^{-1}$.
- Mass can also be slowly accreted onto the surface of the neutron star (or white dwarf) if its magnetic field is weak
- When it releases a critical density the H (and later He) can star burning. T
- Star can increases in luminosity by 10^4 - 10^8 over the period of a few days (declines over period of days-months).
- For a white dwarf the H burning produces a nova. For neutron stars the He burning causes an X-ray burster (repeating X-ray emission).

- **Special Relativity Paradoxes**

- **Narcissistic Runner**

- Imagine a runner (who runs close to the speed of light). He holds a mirror in front of him (at arms length). Will he be able to see himself in the mirror?
- This problem was one of the things Einstein puzzled about as a child.

- **Moving Train**

- Three people (A,B,C) are on a train that is moving near the speed of light. A is at the front, B at the back and C in the middle. A fourth person (C') is standing beside the rail track. At the instant C passes C' they both receive two signals from flash lights that A and B are holding.
- Who sent the signal first ?