Black Holes

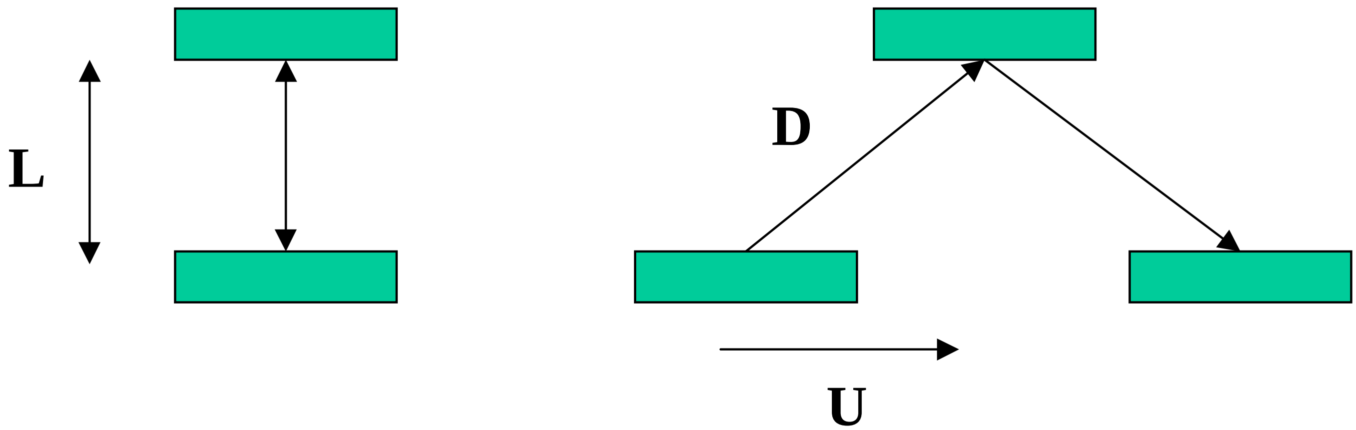
- **Goals:**
  - Why doesn't Newtonian Gravity hold at high velocities?
  - How does Special Relativity help solve these problems.
  - How do we observe black holes.
- **Black Holes**
  - A consequence of gravity
    - Massive neutron (>3$M_\odot$) cannot be supported by degenerate neutron pressure and collapses.
    - The strength of the gravity on the surface of the star increases dramatically.
    - The gravitation fields can no longer be described by Newtonian mechanics (it is not a complete theory).
    - We require a new theory (special and general relativity).
    - These theories do not always appear very intuitive.

- **Why Special Relativity?**
  - **The velocity addition problem.**
    - **Imagine we have 2 cars (A, B)**
    - **In the restframe of car A.**
      - **Car B is moving at a velocity U**
      - **Car B throws a ball out of the window with a velocity V**
    - **In the restframe of car B**
      - **Car A is moving at velocity -U**
      - **Car B throws a ball with velocity V'**
    - **Using Newton's Laws**

      $$V = U + V'$$
  - **In terms of distance and time**
    - **In the restframe of car B**
      - **In time T the ball moves a distance** $X_{ball} = V' T$
    - **In the restframe of car A**
      - **In time T Car B moves a distance** $X_{car\ B} = UT$
      - **The ball moves a total distance** $X = X_{car\ B} + X_{ball} = UT + V'T$
      - **As** $X = VT$ **then** $VT = UT + V'T$
  - **What if car B moves at the speed of light and fires a laser?**

- **Postulates of Special Relativity**
  - **The laws of physics are the same in every inertial frame**
    - **The laws of physics apply in any inertial frame without modification regardless of position or velocity.**
    - **This means both the forms of the laws and the numerical values of the constants. You can not distinguish between inertial frames using the laws of physics.**
  - **Space and time form a 4D continuum**
    - **Space and time used to be considered as separate 3 dimensional and 1 dimensional continua (Galileo).**
    - **This is a generalization of that description.**
    - **It holds for General Relativity as well.**
  - **The speed of light is constant in any inertial frame.**
    - **If we move rapidly (close to the speed of light) light will still have the normal speed of light in our reference frame (e.g. the runner with a mirror experiment).**

- **Time is relative**
  - **Bouncing Light off Mirrors**
    - **Imaging 2 sets of mirrors with light bouncing between them.**



- **Light moves at a constant speed in each inertial frame. Lets consider the passage of time from the stationary point of view.**
- **For the stationary mirrors the time for the reflection of light is given by**

$$T' = \frac{2L}{c}$$

- **The stationary mirror sees the moving mirror as says the time for reflection is**

$$T = \frac{2D}{c}$$

- **The time can only be the same if the distances are the same (clearly not true). But both inertial frames believe they are measuring the same time.**

– Bouncing Light off Mirrors

- From Pythagorus' rule we can measure D

$$D^2 = L^2 + \left(\frac{UT}{2}\right)^2$$

- Substituting for D

$$(cT)^2 = 4\left[L^2 + \left(\frac{UT}{2}\right)^2\right]$$

- Expressing L in terms of T'

$$(cT)^2 = (cT')^2 + (UT)^2$$

- Which simplifies to

$$T = \frac{T'}{\left[1 - \left(\frac{U^2}{c^2}\right)\right]}$$

– The stationary and moving mirrors <u>do not measure the same time</u> !

- This effect is <u>relativistic time dilation</u>. The stationary mirror sees the clock in the moving mirror running slower.
- Time is relative.
- As U→0 (much less than c) T→T'
- Special relativity works for both high and low velocities. It extends Newtonian mechanics.

- **Space is also relative**
  - **Moving cars**
    - **Car B moves at velocity U relative to car A**

    - **Suppose car B measures her cars length as L'. She sees car A moving at velocity -U towards her and it takes time T' for the front of A to pass from the front to the back.**

$$L' = UT'$$

    - **L': Measured length of A (measured by B)**
      T': Time for front to back to pass by
      U: velocity of A relative to B.
    - **A now measures the time T that it takes car B to pass her front bumper.**

$$L = UT$$

    - **We now relate the ratio of lengths to time (which we know from before).**

$$\frac{L}{L'} = \frac{T}{T'}$$

    - **The relation between the distances is then**

$$L = L'\left[1 - \left(\frac{U}{c}\right)^2\right]^{1/2}$$

– The length of the car is relative

- **Note that the time T/T' is the opposite way around than before as the car A is now measuring time relative to a fixed point.**
- **This effect is called <u>relativistic length contraction</u>.**
- **The length of the car B in its rest frame is measured to be much smaller in the rest frame of car A.**
- **Both time and distance are relative (they depend on the inertial frame of the observer). Measuring distances and time intervals is no longer independent of the inertial frame.**

– Lorentz Transformation

- **Combing these effects we get a set of transformations the Lorentz transformations (Hendrik Lorentz) that enable us to move from one inertial frame to the next.**
- **They are derived by requiring that two people (one at rest and another moving) must see an equivalent expanding spherical shell of light.**

- **Examples**
  - **Twin Paradox**
    - **Imagine two twins one sets off in a space ship travelling at 80% the speed of light (to Proxima Centauri). In the space ship frame the twin experiences a travel time (there and back) of**

$$\begin{aligned} T_{space} &= 2\,x\,\frac{4.3\,ly}{0.8} \\ &= 10.75\,yrs \end{aligned}$$

    - **On earth the second twin finds that the space ship clocks have been ticking slower than the earth clocks.**

$$\begin{aligned} T_{earth} &= \frac{T_{space}}{\left[1-(0.8)^2\right]} \\ &= 29.86\,yrs \end{aligned}$$

    - **The twins are now different ages !**
  - **Muon decay**
    - **Protons from interstellar space collide with the atmosphere producing muons. These particles have a decay time of $2.2x10^{-6}$ s and a velocity of 99.9% of light.**
    - **The time to reach the earth is**

$$\begin{aligned} t &= \frac{10{,}000\,m}{299700000\,m\,s^{-1}} \\ &= 3.3x10^{-5}\,s \end{aligned}$$

    - **Yet we detect them!**

– Muon Decay

- If we consider time dilation then the decay time of the muon will increase. The lifetime of the muon as observed from Earth is

$$T_{Earth} = \frac{2.2x10^{-5}}{\sqrt{1-(0.999)^2}}$$
$$= 4.9x10^{-5}\ s$$

- If we consider the distance to the Earth as measured by the muon.

$$L_{Earth} = 10000\ x\ \sqrt{1-(0.999)^2}$$
$$= 447\ m$$

- This takes the muon

$$T_{travel} = \frac{450\ m}{0.999\ x\ 3x10^{8}\ m\ s^{-1}}$$
$$= 1.5x10^{-6}\ s$$

- Relativity has a measurable effect.

• Event Horizon

– Distortion of Spacetime

Figure 24-7, 24-8

- As the neutron star collapses the gravity at the surface of the star increases.
- This causes a distortion in the space and time around the neutron star/black hole.
- The star collapses to a singularity (infinitely small point at the center of the black hole).
- Light now follows a curved path (as opposed to straight lines it usually follows).
- Eventually the escape velocity exceeds the speed of light and light can no longer escape from the star (a black hole).

– Schwarzschild Radius

- For a non-rotating black hole we can calculate the distance to the event horizon.
- Schwarzschild did this by solving the General Relativity equations.
- We define the escape velocity to be

$$\frac{1}{2} m v^2 = \frac{GMm}{R}$$

- v: speed of particle
  m: mass of particle
  M: mass of star/black hole

– Schwarzschild Radius

- The radius at which light does no have enough velocity to escape is

$$R = \frac{2GM}{c^2}$$

  This derivation is wrong but gets the right answer!
- For a $60M_{\odot}$ black hole

$$R = \frac{2 \text{ x } 6.67\text{x}10^{-11} \text{ x } (1.99\text{x}10^{30} \text{ x } 60)}{\left(3\text{x}10^{8}\right)^{2}}$$

$$= 177\text{x}10^{3} \text{ m}$$

- Close than 177 km light cant escape
- Cant transmit information to the outside once inside the event horizon.

– Beyond the Event Horizon

- The strong effects of general relativity only operate locally to the black hole.
- Far from the event horizon Newtonian mechanics works fine.
- Through time dilation an object falling towards the event horizon would appear to approach the event horizon increasing slowly.

– Approaching the Event Horizon

- Far from the event horizon space and time are “flat”. They behave as we would expect from Euclidean mechanics.
- As we approach the event horizon space becomes curved and time slows down (time dilation).
- If we observed a person falling into a black hole as they approached the event horizon their clocks appear to slow down (they never reach the event horizon).
- In their own frame of reference their clocks seem to tick normally (at least for a while).
- The gravitational force is so strong that there are tidal forces between your head and feet. Imagine you are just outside the event horizon of a $60M_{\odot}$ black hole.

$$g_{BH} = \frac{GM_{BH}}{R_{BH}^2}$$

- g_{BH}= surface gravity at event horizon. R_{BH}=Event Horizon

– Approaching the Event Horizon

- **If you were 2m tall then what is the gravitational acceleration of your feet relative to your head.**

$$g_{\text{Feet}} - g_{\text{Head}} = GM_{BH}\left(\frac{1}{R_{BH}^2} - \frac{1}{(R_{BH}+2)^2}\right)$$

$$= 5.7\text{x}10^6 \text{ m s}^{-2}$$

Your feet feel $5.9\text{x}10^5$x the gravitational pull of the Earth. You would be ripped apart by gravitational forces.

– Gravitational Redshift

- **Light escaping from the surface of the BH must overcome this gravitational field. It loses energy. This redshifts the light and is called a <u>gravitational redshift.</u>**

$$z_g = \frac{\lambda - \lambda_o}{\lambda} = \frac{GM}{Rc^2} \qquad \textbf{(Newtonian)}$$

- **z_g: Gravitational redshift**

$$\frac{\lambda}{\lambda_o} = \left[1 - \frac{2GM}{Rc^2}\right]^{-1/2} \qquad \textbf{(GR)}$$

- **For white dwarfs the Newtonian approximation applies ($0.6M_\odot$ WD with $R=0.01R_\odot$ has $z_g=10^{-4}$).**

- **Structure of a Black Hole**
  - **Shielding by the Event Horizon**
    - **As light cant escape from a black hole neither can information (we do not know what a black hole looks like inside nor how it arose).**
    - **We can however measure its mass (Kepler's law), charge (should be minimal) and angular momentum. These numbers characterize a black hole.**
    - **(Note: To describe a star we need to specify its radius, its mass, its composition, its structure etc).**
    - **<u>The event horizon of a rotating black hole is smaller than for a non rotating black hole, but is still spherical in shape</u>. The event horizon is smaller because the inward force of gravity is diminished to some extent by the outward force caused by the spinning.**
    - **The event horizon of a Black Hole rotating at its maximum possible speed is a factor of 2 smaller than for a non-rotating Black Hole.**

– Hawking Radiation

- Due to quantum physical processes a Black Hole has a finite temperature, and thus emits radiation. This radiation ultimately comes from the Black Hole's mass, and so the BH "Evaporates." The evaporation times for solar mass objects is extremely long.
- The BH radiates as a black body with a temperature

$$T = \frac{hc}{8\pi^2 k\, R_S}$$

- h: Planck's constant
  k: Boltzmann's Constant
  R_s:Schwartzschild radius
- The energy radiated comes from the rest mass of the black hole. For a 1 $M_\odot$ BH the temperature is 6×10^{-8} K. As a consequence, a 1 solar mass black hole will take 10^{67} years to disappear. Black holes of larger masses take even longer!

– Observing Black Holes

- **If we were to watch an entire star collapse into a black hole, we would simply see all of its electromagnetic radiation become infinitely redshifted, and then disappear, in a fraction of a second.**
- **At present the only methods used to detect a black hole are indirect. We search for matter orbiting around a candidate BH and identify binary systems which have invisible companion stars with masses greater than $3M_{sun}$.**
- **Extremely hot matter is expected to orbit around a black hole. Matter accreted by a black hole will not fall in but will go into orbit around it (accretion disk). The orbital motions will be so energetic that friction will heat the disk (X-ray emission).**
- **The best stellar black hole candidates are in X-ray binaries where the invisible companion star has a mass greater than $3M_{sun}$. Cygnus X-1 is one of the best candidates.**
- **Gravitational Waves provide a possible means of detecting the formation of a Black Hole.**