Gravitation and the Planets

• **Goals:**
  – How did we discover we orbit the Sun.
  – What laws define the orbits of the planets.
  – What are the laws of Newtonian Mechanics (Gravity).

• **Geocentric Cosmogonies**
  – Does the Sun orbit the Earth?
    
    see Figures 4-2, 4-3
    
  • Greeks believed that the stars were fixed on a rotating celestial sphere.
  • The Sun and moon move in the opposite direction (west-east).
  • The planets (“wanders”) move west-east (“direct”) and east-west (“retrograde”).
  • Ptolemaic system invoked planets rotating in epicycles (small circles) with the center orbiting the earth (a deferent).
  • Mars: retrograde every 22.5 months.
• **Heliocentric Cosmogonies**

  – **No absolute frame of reference**
    • From an object orbiting a planet the planet appears to be orbiting it!
    • E.g. on a train as it moves away from the station - the station appears to move.

  – **Retrograde motion**
    see Figure 4-4
    • Each planet orbits the Sun at a different period - we “over take” the other planets.
    • Retrograde motion comes from our motion.

  – **Copernican Model (1500s)**
    see Figure 4-6
    • The radius of a planet’s orbit and its position along that orbit defines its place on the sky.
    • Planets between Earth and Sun - inferior
    • Planets beyond the Earth’s orbit - superior
– Periods of the planets

• Period is related to the radius of the orbit.

• Synodic period (P, E) - time between 2 identical configurations relative to earth.

• Sidereal period (S) - time for a true orbit.

\[
\text{Earth Orbits at } \frac{360^\circ}{E} \Rightarrow \text{in one Synodic Period } \frac{360^\circ}{E} S
\]

\[
\text{Planet Orbits at } \frac{360^\circ}{P} \Rightarrow \text{in one Synodic Period } \frac{360^\circ}{P} S
\]

**Inferior Planet: Planet “laps” Earth**

\[
\frac{360}{P} S = \frac{360}{E} S + 360 \quad \Rightarrow \quad \frac{1}{P} = \frac{1}{E} + \frac{1}{S}
\]

**Superior Planet: Planet “lags” Earth**

\[
\frac{360}{P} S = \frac{360}{E} S - 360 \quad \Rightarrow \quad \frac{1}{P} = \frac{1}{E} - \frac{1}{S}
\]

• Jupiter: \( S = 398.9 \text{ days} = 1.092 \text{ yrs} \)

\( P \text{ (Sidereal)} = 11.87 \text{ yrs} \)
New Observations of the Planets

• Tycho Brahe
  – Parallax
    • If an observer moves then foreground objects change relative to the background.
    • Closing each eye changes the image you see (e.g. a pen in front of your face).
    • The movement of the foreground object is greater the closer it is to you.
    • The rotation and orbit of the earth can cause this effect.
  – New observations
    • High precision (1 arcmin) positions of the stars.
    • Could not measure parallax - believed the Earth was stationary (parallax of nearby stars is < 1 arcsec).

• Johannes Kepler
  – New model for the orbit of the planets
    • Motion of planets are described by ellipses
– **Ellipses**
  
  • Circle has one focus - ellipse has 2 foci.
  • Shape of the ellipse is defined by the semi-major (a) and semi-minor (b) axes.
  • Circle is a special case of an ellipse.
  • Elliptical orbits match the data precisely.

– **Kepler’s First law**
  
  • Orbit of a planet about the Sun is an ellipse with the Sun at one focus.
  • The length of the semi-major axis is the average distance from the planet to the Sun.
  • Ellipse shape given by eccentricity \( e \)
    \[
    e = \frac{\sqrt{a^2 - b^2}}{a}
    \]
    
    Venus \( e=0.007 \)  
    Pluto \( e=0.248 \)  
    (circle) \( 0 < e < 1 \)  
    (line)
  
  • Perihelion (closest to Sun): Planet moves rapidly.
  • Aphelion (furthest from Sun): Planet moves less rapidly.
- **Kepler’s Second Law**

  **Figure 4-11**
  
  - A line joining a planet and the Sun sweeps out the equal areas in equal amounts of time.
  
  - A circle is an idealized case of this.

- **Kepler’s Third Law**

  - The square of the **sidereal** period of a planet is directly proportional to the cube of the semi-major axis of the orbit.

  \[ P^2 = a^3 \]

  - P (Sidereal Period) measured in **years**
  
  - a (semi-major axis) measured in **AU**

  - Larger the orbit, longer the period and slower the average speed.

  - Elliptical orbit with variable speed removes the need for retrograde corrections.

Kepler discovered these relations empirically

A fundamental result
• **Galileo Galilei**

  – **Verified Kepler's results with new observations**
    
    • Orbits of the moons around Jupiter.
    • Phases of Venus.
    • Rings of Saturn.
    • Sunspots.
    • Mountains on the moon.

  – **Each observation helped improve the validity of the Copernican model**

    **Figures 4-14, 4-15**

    • Variable size of Venus (and its relation to its phases) cannot be explained by the Ptolemaic model.
    • Period of the moons of Jupiter depended on their semi-major axis.
    • A paradigm shift.
Newtonian Mechanics

• Newton’s first Law (law of inertia)
  – A body remains at rest or moves in a straight line at constant speed unless acted upon by a net outside force.
    • Net force = combined effect of all outside forces (vector sum).
    • With no gravity planets would “fly off”. Gravity maintains the orbits.
    • Law is related to inertia (we stay in the same state unless acted upon).

• Newton’s Second Law
  – Acceleration of an object is directly proportional to the net outside force.

\[ F = m \ a \]

F: force
m: mass
a: acceleration
• **Newton’s Second Law**

  – **Acceleration: rate of change of velocity**
    - Acceleration can change the **speed** or **direction** of an object.
    - Planets are accelerated by **Gravity** (changing their direction).
    - Acceleration occurs in the direction of the **force**.
    - Force must overcome inertia to accelerate.

  – **Example**
    - Earth’s surface gravity is 9.8 m/s².
    - A 50 kg person jumps from a plane.
      
      \[
      F = ma \\
      = 50 \times 9.8 = 490 \text{ Newtons} = 110 \text{ pounds}
      \]
    - On the moon (g = 1.666 m/s²).
      \[
      F = 50 \times 1.666 = 83.3 \text{ Newtons} = 18.7
      \]
    - Weight (Force) ≠ Mass (Inertia)
• **Newtons Third Law**

  - When one body exerts a force on another body the second body exerts an equal and opposite force on the first body.
  
    • Equal and opposite forces act on different bodies.

  - **Example:**

    • A shotgun - pellets and the gun receive the same force. The mass of the pellets is much smaller so they accelerate faster.
    
    • mass of shot = 1oz = 0.025 kg.
    
    • mass of person = 50 kg.
    
    \[
    F_{\text{shot}} = F_{\text{person}}
    \]
    
    \[
    a_{\text{shot}} = \frac{m_{\text{person}}}{m_{\text{shot}}} a_{\text{person}}
    \]
    
    \[
    a_{\text{shot}} = 2000 a_{\text{person}}
    \]
    
    • Sun exerts a force on the planets and they exert an equal and opposite force of the Sun (Earth is 1/300,000 mass of Sun: acceleration of the Sun is small).
• **Gravitation and the Planets**
  
  – **Gravity shapes the orbits**
    
    • The pull of gravity makes planets “fall” towards the Sun (same as the pull of the earth on an apple).
    
    • The smaller the semi-major axis - faster rotation, requires more force.

• **Newton’s Law of Gravity**

  – Two bodies attract each other with a force that is directly proportional to the mass of each body and inversely proportional to the square of the distance between them.

\[
F = \frac{G m_1 m_2}{r^2}
\]

  
  F: Force
  
  \(m_1\): mass of object 1
  
  \(m_2\): mass of object 2
  
  \(r\): distance between objects
  
  \(G\): Gravitational constant (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}

  • Double distance: quarter force.
  
  • Double mass: double force.
– **Example:**

- Sun-Earth **and** Earth-Sun.
  \[ M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg} \]
  \[ M_{\text{earth}} = 5.98 \times 10^{24} \text{ kg} \]
  \[ r = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m} \]

\[
F_{S-E} = 6.67 \times 10^{-11} \frac{5.98 \times 10^{24} \times 1.99 \times 10^{30}}{(1.5 \times 10^{11})^2}
\]

\[ F_{S-E} = 3.53 \times 10^{22} \text{ N} \]

– **Why do we orbit not fall into the Sun?**

**Figure 4-20**

- Tangential motion stops the Earth falling directly towards the Sun.
- If speed is right curvature of the path matches curvature of Sun (circular orbit).
- If the speed is greater or slower (a little) the orbit becomes elliptical.
- If speed is much greater we escape from orbit (hyperbolic or parabolic curve).
- The centripetal acceleration \( a_r \) required to keep the ball in orbit with velocity \( v \) is
  \[ a_r = \frac{v^2}{r} \]
- Using \( F=ma \) implies that the orbital velocity is given by
  \[ v = \left( \frac{GM}{r} \right)^{1/2} \]
– **Surface Gravity (g).**

  • A measure of the acceleration of an object at the planet (or stars) surface.

  • From Newtons Law of Gravity the acceleration is independent of the mass of the object, hence given special symbol g.

  • M= Mass of planet, R= Radius of planet.

  • g = GM / R²

    – g=9.8 m/s² for Earth.

    – g for the Moon is a factor of 6 smaller than g for the Earth.

– **Conservation of Energy**

  • The total energy of an enclosed system is constant. Energy may be converted from one form to another but not created or destroyed.

  • Energy --- the ability to do work.

  • Kinetic energy --- energy of motion = $E_k = 1/2 m v^2$

  • Potential Energy --- $E_p = -GMm/r$
– Conservation of Momentum

• Total momentum of particles in a system remains unchanged.
• Linear momentum
  – \( = m \mathbf{v} \) (vector).
  – Linear momentum is constant unless acted on by an EXTERNAL force.
• Angular momentum (L).
  – The angular momentum (L) of an object moving with velocity \( V \) with respect to an axis at distance \( r \) is
  – \( L = m \mathbf{r} \times \mathbf{v} \) (cross product)
  – Kepler’s 2nd law is simply conservation of Angular momentum.
– **Escape velocity**

- A measure of the velocity required by an object to escape the gravitational pull of the planet (or star). The escape velocity, $V_{esc}$, is defined such that it is the *minimum* speed an object needs so that it will never fall back to the planets surface.

- At infinity $E_k + E_p = 0$, since object just makes it to infinity.

- As object is fired from the Earth
  - $E_k = \frac{1}{2} m v^2$
  - $E_p = -\frac{GMm}{r}$

- Since energy is conserved we have $E_k + E_p = 0$ hence
  - $\frac{1}{2} m v^2 = \frac{GMm}{r}$

- Thus (for a spherical Earth)
  - $V_{esc} = \left( \frac{2GM}{R} \right)^{1/2}$

- where $M$ is the Mass of the planet, and $R$ its radius.
• **Newton’s form of Kepler’s law**

\[
P^2 = \frac{4\pi^2}{G(m_1 + m_2)} a^3
\]

\(a\): semi-major axis (average distance).

– **Many applications**
  • Measure the masses of binary stars.

• **Progress of Science**
  – **Kepler: New observations**
  – **Newton: New model/theory**
    • Theory predicted Halleys comet’s return and was used in the discovery of Neptune.
    • Cornerstone of modern physics.