Practice Final

Note: **THIS TEST IS LONGER THAN THE ACTUAL TEST.** ***It is a sample and does not include questions on*** ever topic covered since the start of the semester.

Also be sure to review
homework assignments on WebAssign
Whiteboard problems worked out in class
Problems from tests 1 - 3
Exercises, Examples, and Review Questions (at the end of each chapter) in your textbook

Some general rules:

- **Read all problems carefully before attempting to solve them**
- **Your work must be legible, and the organization must be clear**
- **You must show all your work, including correct vector notation**
- **Correct answers without adequate explanation will be counted as wrong**
- **Incorrect work or explanations mixed in with correct work will be counted as wrong**
    - **Cross out anything you don't want us to read!**
- **Make explanations complete but brief. Do not write a lot of prose.**
- **Include diagrams!**
- **Show what goes into a calculation, not just the final number:**
  $$\frac{a \cdot b}{c \cdot d} = \frac{\left(8 \times 10^{-3}\right)\left(5 \times 10^{6}\right)}{\left(2 \times 10^{-5}\right)\left(4 \times 10^{4}\right)} = 5 \times 10^{4}$$
- **Give standard SI units with your results**

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.

Problem 1. A baseball of mass 0.025 kg is released from rest at a location 3.5 m above the ground. You will calculate its speed at the instant it hits the ground.

(a) Choose a system to analyze, and list the objects in your chosen system.

We can make a few choices. I will choose the baseball to be the system, so the only object in the system is the baseball.

(b) Write out the Energy Principle *as it applies to the system you have chosen.* Solve for the speed of the baseball when it hits the ground.

The Energy Principle in general is $\Delta E = W + Q$. We have a very small $Q \approx 0$, so we can write this as

$$E_f = E_i + W\ .$$

The only energy that is changing in the system is kinetic (ignoring air resistance, which is very small since the ball will be moving slowly), so

$$K_f = K_i + W\ .$$

The initial kinetic energy is $K_i = 0$ since the ball is dropped from rest. The final kinetic energy is $K_f = \frac{1}{2}mv^2$. Thus solving for the velocity, we have

$$v = \sqrt{\frac{2W}{m}}\ .$$

The work is done by gravity, which is a constant force of magnitude $F = mg$. Work is

$$W = \vec{F} \cdot \Delta\vec{r}$$

which in this case is

$$W = mgh,$$

where $h = 3.5\ \text{m}$. Putting all this back together, we have

$$v = \sqrt{\frac{2mgh}{m}} = \sqrt{2gh} = \sqrt{2 \times 9.8\ \text{m/s}^2 \times 3.5\ \text{m}} = \boxed{8.28\ \text{m/s}}$$

Problem 2. In transitions between vibrational energy levels of the diatomic molecule HI (hydrogen iodide), photons are emitted with energy 4.4e-20 J, twice this energy, three times this energy, etc. Because the mass of the iodine nucleus is much larger than that of the hydrogen nucleus, it is a good approximation to consider the iodine nucleus to be always at rest; you can think of the H atom as being attached by a "spring" to a wall. What is the effective stiffness of the HI bond? Show your work.

The vibrational energy levels for a harmonic oscillator (mass connect to a spring) are

$$\Delta E = \hbar\omega .$$

We also know that

$$\omega = \sqrt{\frac{k_s}{m}},$$

which means

$$\Delta E = \hbar\sqrt{\frac{k_s}{m}} \quad \Rightarrow \quad k_s = m\left(\frac{\Delta E}{\hbar}\right)^2 .$$

Plugging in numbers for this case,

$$k_s = 1.7\times10^{-27}\ \text{kg}\left(\frac{4.4\times10^{-20}\ \text{J}}{1.05\times10^{-34}\ \text{J}\cdot\text{s}}\right)^2 = \boxed{299\ \text{kg/s}^2}.$$

Problem 3. The neutron has a mass of 1.6749e-27 kg. The neutron is unstable when removed from a nucleus, and it decays into a proton, electron and antineutrino. The proton's mass is 1.6726e-27 kg, the electron's mass is 9.1e-31 kg, and the antineutrino has nearly zero mass. Assume that the original neutron was at rest. What is the total kinetic energy of the proton plus electron plus antineutrino when they are far apart? Show all work.

This uses the Energy Principle, $\Delta E = W + Q$. We will take as our system all the particles. There are no external forces, so the work is zero. Also $Q = 0$, so we have

$$E_f = E_i .$$

Writing out the energies, we have

$$m_p c^2 + K_p + m_e c^2 + K_e + m_v c^2 + K_v = m_n c^2 + K_{n.}$$

The neutrino is essentially massless, and the neutron was initially at rest, so

$$m_p c^2 + K_p + m_e c^2 + K_e + K_v = m_n c^2 .$$

Solving for the sum of the final kinetic energies

$$K_p + K_e + K_v = m_n c^2 - m_p c^2 - m_e c^2 = \left(m_n - m_p - m_e\right)c^2 .$$

Plugging in numbers

$$K_p + K_e + K_v = \left(1.6749\times10^{-27} - 1.6726\times10^{-27} - 9.1\times10^{-31}\right)\text{ kg}\times\left(3\times10^8\text{ m/s}\right)^2 = \boxed{1.25\times10^{-13}\text{ J}}.$$

This corresponds to 0.78 MeV.

Problem 4. You apply a force of <10, -25, 8> N on an object while the object moves from position <-4, 3, 6> m to position <-1, 6, 2> m. How much work do you do?

Work is $W = \vec{F} \cdot \Delta\vec{r}$. In this case,

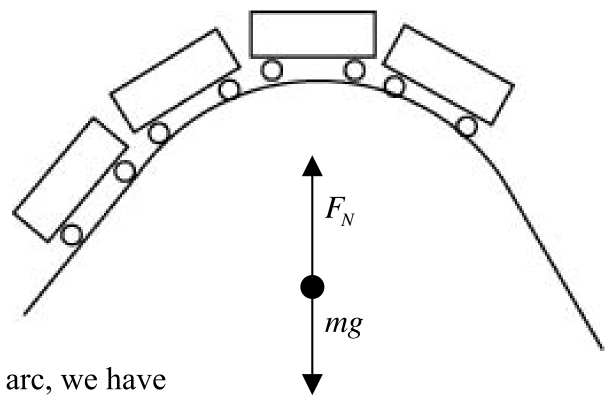
$$\Delta\vec{r} = \langle -1,6,2 \rangle \text{ m} - \langle -4,3,6 \rangle \text{ m} = \langle 3,3,-4 \rangle \text{ m.}$$

So

$$W = \langle 10,-25,8 \rangle \text{ N} \cdot \langle 3,3,-4 \rangle \text{ m} = \left(30 - 75 - 32\right) \text{ J} = \boxed{-77 \text{ J}}.$$

Problem 5. By "weight" we usually mean the gravitational force exerted on an object by the Earth. However, when you sit in a chair your own perception of your own "weight" is based on the contact force the chair exerts upward on your rear end rather than on the gravitational force. The smaller this contact force is, the less "weight" you perceive, and if the contact force is zero, you feel peculiar and "weightless" (an odd word to describe a situation when the only force acting on you is the gravitational force exerted by the Earth!). Also, in this condition your internal organs no longer press on each other, which presumably contributes to the odd sensation in your stomach.

(a) How fast must a roller coaster car go over the top of a circular arc for you to feel "weightless"? The center of the car moves along a circular arc of radius $R = 20$ m. Include a carefully labeled force diagram.

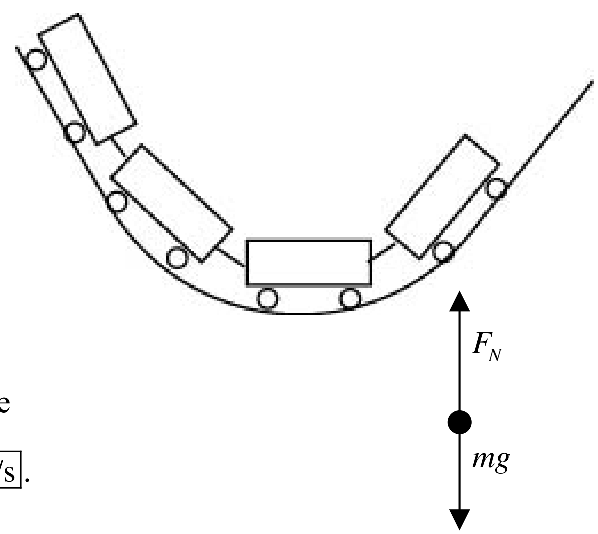


My force diagram is shown to the right.

The net force is therefore $F_{\text{net}} = F_N - mg$ in the vertical direction. According to the Momentum Principle, we have $\Delta\vec{p} = \vec{F}_{\text{net}}\Delta t$, or since we are going around a circular arc, we have $|\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \vec{F}_\perp$. Simplifying this a bit, we have $\frac{mv^2}{R} = \left|\vec{F}_{\text{net}}\right| = \left|F_N - mg\right|$. We therefore must have $\frac{mv^2}{R}$ equal mg so that $F_N = 0$. In other words,

$$v = \sqrt{Rg} = \sqrt{20\text{ m} \cdot 9.8\text{ m/s}^2} = \boxed{15\text{ m/s}}.$$

(b) How fast must a roller coaster car go through a circular dip for you to feel three times as "heavy" as usual, due to the upward force of the seat on your bottom being three times as large as usual? The center of the car moves along a circular arc of radius $R = 20$ m. Include a carefully labeled force diagram.



In this case, we have a similar force diagram. However, we now need the normal force to be $3mg$, which means $F_{\text{net}} = 3mg - mg = 2mg$. Setting this equal to $\frac{mv^2}{R}$, we have

$$v = \sqrt{2Rg} = \boxed{19.8\text{ m/s}}.$$

Problem 6. An electron is moving with constant speed $0.97c$, where c is the speed of light.

(a) What is its rest energy?

The rest energy is

$$E_{\text{rest}} = m_e c^2 = 9\times10^{-31}\ \text{kg}\ \left(3\times10^8\ \text{m/s}\right)^2 = \boxed{8.1\times10^{-14}\ \text{J}}.$$

This corresponds to 0.51 MeV.

(b) What is its total energy?

The total energy is

$$E_{\text{total}} = \gamma m_e c^2 = \frac{1}{\sqrt{1-0.97^2}} m_e c^2 = 4.11 m_e c^2 = \boxed{3.3\times10^{-13}\ \text{J}}.$$

This corresponds to 2.1 MeV.

(c) What is its kinetic energy?

The kinetic energy is

$$K = E_{\text{total}} - E_{\text{rest}} = (\gamma-1) m_e c^2 = 3.11 m_e c^2 = \boxed{2.5\times10^{-13}\ \text{J}}.$$

This corresponds to 1.6 MeV.

Problem 7. At $t = 532.0$ s after midnight a spacecraft of mass 1400 kg is located at position <3e5, 7e5, -4e5> m, and an asteroid of mass 7e15 kg is located at position <9e5, -3e5, -12e5> m. There are no other objects nearby.

(a) Calculate the (vector) force acting on the spacecraft.

The force is due to gravity, $\vec{F}_{\text{grav}} = -G\dfrac{m_1 m_2}{r^2}\hat{r}$. So first, let's calculate the vector $\vec{r}$ from the asteroid to the spacecraft:

$$\vec{r} = \langle 9\text{e}5, \text{-}3\text{e}5, \text{-}12\text{e}5\rangle \text{ m} - \langle 3\text{e}5, 7\text{e}5, \text{-}4\text{e}5\rangle \text{ m} = \langle 6\text{e}5, \text{-}10\text{e}5, \text{-}8\text{e}5\rangle \text{ m}.$$

The magnitude of $\vec{r}$ is

$$|\vec{r}| = \sqrt{(6\text{e}5)^2 + (\text{-}10\text{e}5)^2 + (\text{-}8\text{e}5)^2} \text{ m} = 14.1\times 10^5 \text{ m}.$$

The unit vector in the $\vec{r}$ direction is

$$\hat{r} = \frac{\vec{r}}{r} = \frac{\langle 6\text{e}5, \text{-}10\text{e}5, \text{-}8\text{e}5\rangle \text{ m}}{14.1\text{e}5 \text{ m}} = \langle 0.43, -0.71, -0.57\rangle.$$

Therefore, the force on the spacecraft is

$$\vec{F}_{\text{on spacecraft}} = -6.7\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2 \frac{1400 \text{ kg}\cdot 7\times 10^{15} \text{ kg}}{(14.1\times 10^5 \text{ m})^2}\langle 0.43, -0.71, -0.57\rangle$$

$$= \boxed{\langle -1.4\times 10^{-4}, 2.3\times 10^{-4}, 1.9\times 10^{-4}\rangle \text{ N}}.$$

(b) At $t = 532.0$ s the spacecraft's momentum was $\vec{p}_i$. At $t = 532.4$ s, the spacecraft's momentum was $\vec{p}_f$. Calculate the (vector) change in momentum $\vec{p}_f - \vec{p}_i$.

Here we use the Momentum Principle, $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \vec{F}_{\text{net}}\Delta t$. We just calculated the force due to gravity. That is the only force acting on the spacecraft. We also have

$$\Delta t = t_f - t_i = 532.4 \text{ s} - 532.0 \text{ s} = 0.4 \text{ s}.$$

Therefore,

$$\Delta\vec{p} = \langle -1.4\times 10^{-4}, 2.3\times 10^{-4}, 1.9\times 10^{-4}\rangle \text{ N}\cdot 0.4 \text{ s} = \boxed{\langle -5.7\times 10^{-5}, 9.4\times 10^{-5}, 7.5\times 10^{-5}\rangle \text{ kg}\cdot\text{m/s}}.$$

Problem 8.

(a) A package of mass m sits on an airless asteroid of mass M and radius R. We want to launch the package straight up in such a way that its speed drops to zero when it is a distance $4R$ from the center of the asteroid, where it is picked up by a waiting ship before it can fall back down. We have a powerful spring whose stiffness is k_s. How much must we compress the spring? Show your work.

We will use the Energy Principle, $E_f = E_i + W + Q$, picking the system to be the package and the asteroid. In that case, $W = Q = 0$, and we have $E_f = E_i$. The initial energy is

$$E_i = K_{\text{asteroid},\,i} + K_{\text{package},\,i} + U_{\text{grav},\,i} + U_{\text{spring},\,i} = 0 + 0 - G\frac{mM}{R} + \frac{1}{2}k_s s^2 .$$

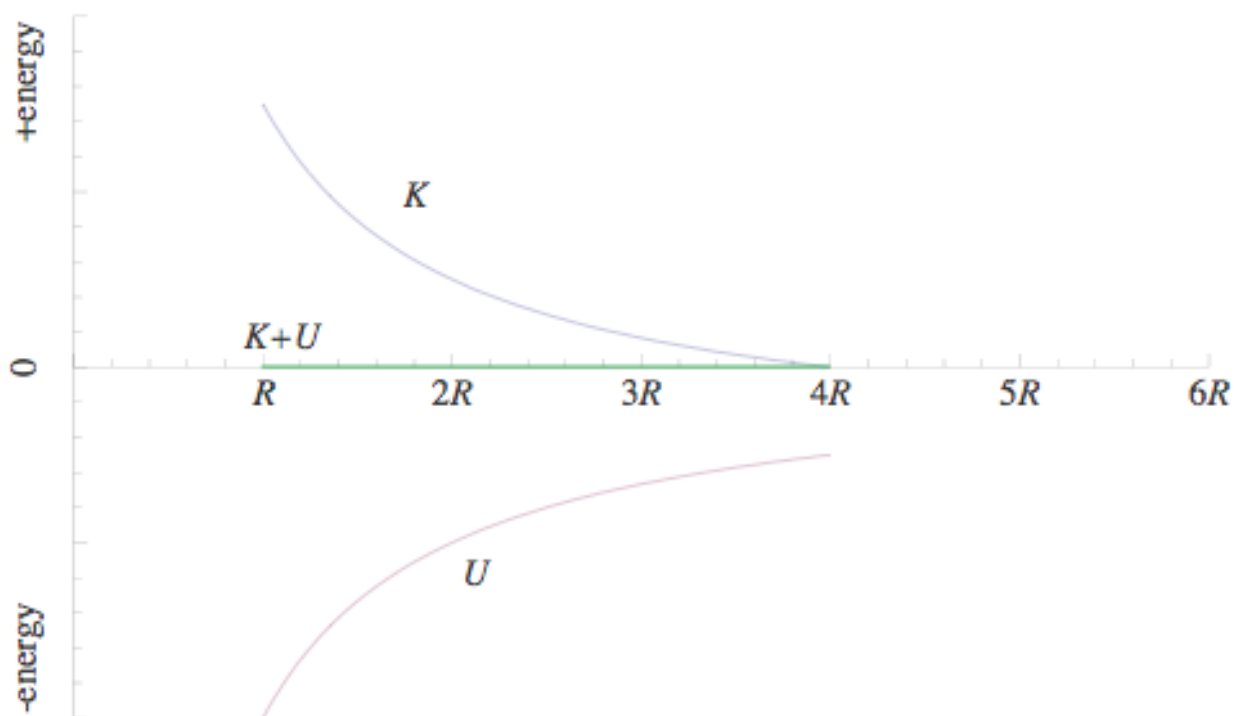
The final energy is

$$E_f = K_{\text{asteroid},f} + K_{\text{package},f} + U_{\text{grav},f} + U_{\text{spring},f} = 0 + 0 - G\frac{mM}{4R} + 0 .$$

Setting the two equal, and solving for s, we have

$$s = \sqrt{\frac{2GmM}{R}\left(1 - \frac{1}{4}\right)} = \boxed{\sqrt{\frac{3GmM}{2R}}}.$$

(b) Starting just after the launch, for the system consisting of the asteroid plus package, graph the gravitational potential energy U, the kinetic energy K and the sum $K+U$, as a function of the distance between the centers of the asteroid and package. Label each curve clearly.



Problem 9. One mole of nickel (6×10^{23} atoms) has a mass of 59 grams, and its density is 8.9 grams per cubic centimeter. You have a long thin bar of nickel, 2.2 m long, with a square cross section, 0.1 cm on a side.

(a) You hang the rod vertically and attach a 104 kg mass to the bottom, and you observe that the bar becomes 1.12 cm longer. Calculate the effective stiffness of the interatomic bond, modeled as a "spring".

We first calculate Young's modulus given the above information:

$$Y = \frac{F/A}{\Delta L/L} = \frac{104\ \text{kg}\cdot 9.8\ \text{m/s}^2 / \left(0.1\times10^{-2}\ \text{m}\right)^2}{1.12\times10^{-2}\ \text{m}/2.2\ \text{m}} = 2.0\times10^{11}\ \text{N/m}^2.$$

To relate this to the interatomic bond effective stiffness, we will use

$$Y = \frac{k_s}{d} \quad\Rightarrow\quad k_s = Yd,$$

where d is the distance between the atoms. So we need to find d. We can use

$$\rho = \frac{m_{\text{atom}}}{d^3} = \frac{M_{\text{per mole}}/N_A}{d^3},$$

so

$$d = \sqrt[3]{\frac{M_{\text{per mole}}/N_A}{\rho}} = \sqrt[3]{\frac{59\ \text{g/mol}/\left(6\times10^{23}\ \text{mol}^{-1}\right)}{8.9\ \text{g/cm}^3}} = 2.23\times10^{-8}\ \text{cm} = 2.23\times10^{-10}\ \text{m}.$$

Therefore,

$$k_s = 2.0\times10^{11}\ \text{N/m}^2\cdot 2.23\times10^{-10}\ \text{m} = \boxed{44.6\ \text{N/m}}.$$

(b) Next you remove the 104 kg mass, place the rod horizontally, and strike one end with a hammer. How much time Δt will elapse before a microphone at the other end of the bar will detect a disturbance?

For this, we need the speed of sound in the metal, which is given by

$$v = d\sqrt{\frac{k_s}{m}}.$$

We already calculated k_s, d, and m is

$$m = \frac{M_{\text{per mole}}}{N_A} = \frac{0.059\ \text{kg/mol}}{6\times10^{23}\ \text{mol}^{-1}} = 9.83\times10^{-26}\ \text{kg}.$$

The speed of sound is then

$$v = 2.23\times10^{-10}\ \text{m}\sqrt{\frac{44.6\ \text{N/m}}{9.83\times10^{-26}\ \text{kg}}} = 4750\ \text{m/s}.$$

The time it takes is therefore

$$\Delta t = \frac{\Delta x}{v} = \frac{2.2\ \text{m}}{4750\ \text{m/s}} = \boxed{4.6\times10^{-4}\ \text{s}}.$$

Problem 10. A block of mass 0.07 kg is attached to the end of a vertical spring whose relaxed length is 0.23 m. When the block oscillates up and down in the lab room, it takes 1.29 s to make a round trip.

(a) What is the stiffness of the spring?

Here we use $\omega = \sqrt{k_s / m} \Rightarrow k_s = m\omega^2$. The angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.29 \text{ s}} = 4.87 \text{ s}^{-1}.$$

Therefore,

$$k_s = 0.07 \text{ kg} \cdot \left(4.87 \text{ s}^{-1}\right)^2 = \boxed{1.66 \text{ N/m}}.$$

(b) At the bottom of the oscillation, when the momentum is momentarily zero, the length of the spring is 0.80 m. What is the net (vector) force acting on the block at this instant?

At the bottom, there is the force of gravity pulling down, and the spring pulling up. As usual, we will make $\hat{y}$ point up. The net force is

$$\vec{F}_{\text{net}} = \left(k_s s - mg\right)\hat{y} = \left(1.66 \text{ N/m}\left(0.80 \text{ m} - 0.23 \text{ m}\right) - 0.07 \text{ kg} \cdot 9.8 \text{ m/s}^2\right)\hat{y} = \boxed{0.26 \text{ N } \hat{y}}.$$

(c) Air resistance eventually damps out the oscillation and the block hangs motionless from the spring. How long is the spring now? Justify your analysis by starting from a fundamental physics principle. No credit will be given without this justification.

We will start with the Momentum Principle, $\Delta\vec{p} = \vec{F}_{\text{net}}\Delta t$. Since the block is now motionless, $\Delta\vec{p} = 0$, and so

$$\vec{F}_{\text{net}}\Delta t = 0 \quad \Rightarrow \quad \vec{F}_{\text{net}} = 0.$$

The net force is

$$\vec{F}_{\text{net}} = \left(k_s s - mg\right)\hat{y},$$

which means that

$$k_s s - mg = 0,$$

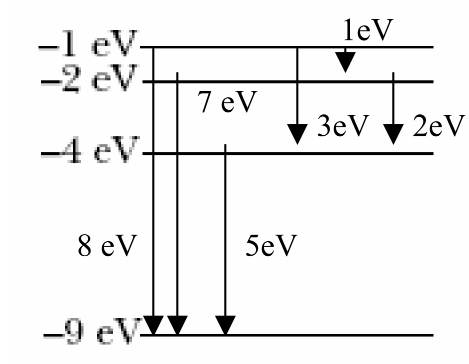
or

$$s = L - L_0 = \frac{mg}{k_s} = \frac{0.07 \text{ kg} \cdot 9.8 \text{ m/s}^2}{1.66 \text{ N/m}} = 0.41 \text{ m}.$$

Solving for the new length,

$$L = L_0 + 0.41 \text{ m} = 0.23 \text{ m} + 0.41 \text{ m} = \boxed{0.64 \text{ m}}.$$

Problem 11. A particular quantum system has energy levels $K+U$, shown in the diagram. A beam of high-energy electrons runs through a bottle that contains a large number of these systems, so that there are many systems in all of these energy states at all times, and photons are continually emitted from the bottle.



(a) What are the energies of the emitted photons? Indicate the transitions on the diagram.

The energies are 8 eV, 7 eV, 5 eV, 3 eV, 2 eV, and 1 eV.

(b) Next the electron beam is turned off, and a beam of light with a wide range of energies, from 0.1 to 15 eV, shines through the bottle. On the other side of the bottle, what photon energies in the beam are somewhat depleted (dark spectral lines)?

The dark lines all start from the ground state, so we have 8eV, 7eV, and 5eV dark lines.

Problem 12. In outer space two rocks collide and stick together, without rotation. Here are the masses and initial velocities of the two rocks:

Rock 1: mass = 15 kg, initial velocity = <10, -30, 0> m/s
Rock 2: mass = 32 kg, initial velocity = <15, 12, 0> m/s

(a) What is the (vector) velocity of the stuck-together rocks after the collision?

We pick the two rocks as the system. There are no external forces, so the Momentum Principle tells us $\Delta\vec{p} = \vec{F}_{\text{net}}\Delta t = 0$. Our initial momentum is

$$\vec{P}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = 15\ \text{kg}\langle 10, \text{-}30, 0\rangle \text{m/s} + 32\ \text{kg}\langle 15, 12{,}0\rangle\ \text{m/s} = \langle 630{,}-66{,}0\rangle\ \text{kg}\cdot\text{m/s}.$$

Our final momentum is

$$\vec{P}_f = (m_1 + m_2)\vec{v} = 47\ \text{kg}\ \vec{v}.$$

Setting the two equal, we have

$$\vec{v} = \frac{\langle 630{,}-66{,}0\rangle\ \text{kg}\cdot\text{m/s}}{47\ \text{kg}} = \boxed{\langle 13.4{,}-1.4{,}0\rangle\ \text{m/s}}.$$

(b) Calculate the increase in thermal energy of the rocks.

From the Energy Principle, $\Delta E = W + Q$, we have

$$E_f = E_i + W + Q = E_i,$$

since $W = Q = 0$ in this case. The energies (ignore rest mass energies) are

$$E_i = K_i + E_{\text{int},i} + U, \qquad E_f = K_f + E_{\text{int},f} + U.$$

The potential U does not change, so we have

$$\Delta E_{\text{int}} = -\Delta K.$$

The initial kinetic energy is

$$K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}15\ \text{kg}\cdot 1000\ \text{m}^2/\text{s}^2 + \frac{1}{2}32\ \text{kg}\cdot 369\ \text{m}^2/\text{s}^2 = 13404\ \text{J}.$$

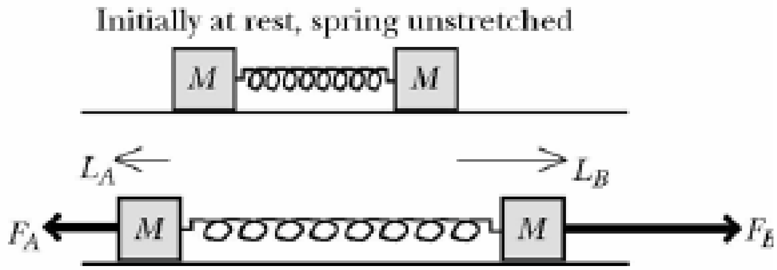
The final kinetic energy is

$$K_f = \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}47\ \text{kg}\cdot 181.5\ \text{m}^2/\text{s}^2 = 4265\ \text{J}.$$

Therefore, the change in thermal energy is

$$\Delta E_{\text{thermal}} = K_i - K_f = 13404\ \text{J} - 4265\ \text{J} = \boxed{9139\ \text{J}}.$$

Problem 13. Here are two equal blocks, each of mass M, connected by a spring. Initially the blocks are at rest and the spring is relaxed. Then Ann starts pulling to the left with a force of magnitude F_A, and Ben starts pulling to the right with a force of larger magnitude F_B. A short time later the left block has moved to the left a distance L_A, and the right block has moved to the right a larger distance L_B. Note that the center of mass of the two blocks has moved to the right a distance $(L_B - L_A)/2$. There is negligible friction with the table.



(a) Use the point particle system to determine the translational kinetic energy of the system consisting of the two blocks and nearly massless spring.

The center of mass moves a distance $(L_B - L_A)/2$, and the net force acting on the point particle system is $F_{\text{net}} = F_B - F_A$ in the same direction as the movement of the center of mass. The initial energy of the point particle system is $E_i = K_i = 0$. Therefore, using the Energy Principle,

$$E_f = K_f = E_i + W = 0 + W = \boxed{\frac{(F_B - F_A)(L_B - L_A)}{2}}.$$

(b) Use the real system to determine the change in the internal energy of the system consisting of the two blocks and the spring, $\Delta K_{\text{rel}} + \Delta U_{\text{spring}}$, where K_{rel} is the kinetic energy of the two blocks relative to the center of mass. Note that the stiffness of the spring is not known, so your result should not contain the spring stiffness. Don't bother simplifying your final result, which should contain only the given quantities (M, F_A, F_B, L_A, L_B).

The Energy Principle tells us

$$\Delta E = \Delta K_{\text{trans}} + \Delta K_{\text{rel}} + \Delta U_{\text{spring}} = W + Q = W,$$

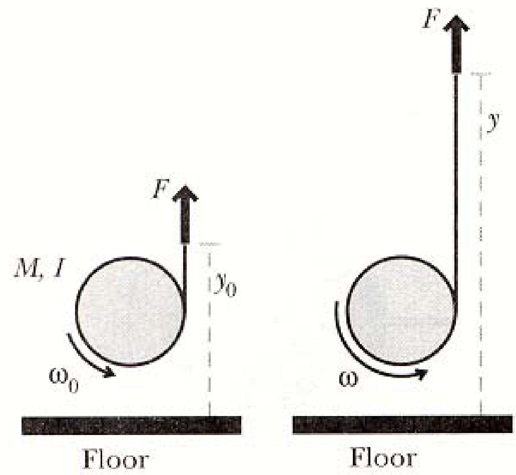
ignoring the possibility of a small heat transfer. The work done on the system is

$$W = F_A L_A + F_B L_B.$$

Therefore,

$$\Delta K_{\text{rel}} + \Delta U_{\text{spring}} = W - \Delta K_{\text{trans}} = F_A L_A + F_B L_B - \frac{(F_B - F_A)(L_B - L_A)}{2} = \boxed{\frac{(F_B + F_A)(L_B + L_A)}{2}}.$$

Problem 14. A string is wrapped around an object of mass $M = 1.2$ kg and moment of inertia $I = 0.06\ \mathrm{kg \cdot m^2}$. You pull the string with your hand straight up with some constant force F such that the center of the object does not move up or down, but the object spins faster and faster. This is like a yo-yo; nothing but the vertical string touches the object. When your hand is a height $y_0 = 0.25$ m above the floor, the object has an angular speed $\omega_0 = 12$ radians/s.



When your hand has risen to a height $y = 0.66$ m above the floor, what is the angular speed ω of the object? Your answer must be numeric and not contain the symbol F.

We will do this problem using the Energy Principle, $\Delta E = W + Q$, picking the object to be the real system (and we will also consider the point particle system). In this case, $Q = 0$. The change in energy can be written as

$$\Delta E = \Delta K_{\mathrm{trans}} + \Delta K_{\mathrm{rot}}.$$

By looking at the point particle system, we see that the center-of-mass does not move, so

$$\Delta K_{\mathrm{trans}} = 0.$$

We also know that, since the momentum of the point particle does not change

$$\frac{\Delta \vec{p}}{\Delta t} = \vec{F}_{\mathrm{net}} = (F - mg)\vec{y} = 0 \quad \Rightarrow \quad F = mg.$$

Looking at the Energy Principle again, we have

$$\Delta K_{\mathrm{rot}} = W.$$

The work done is

$$W = F(y - y_0) = mg(y - y_0).$$

Expanding the change in rotational kinetic energy in the above,

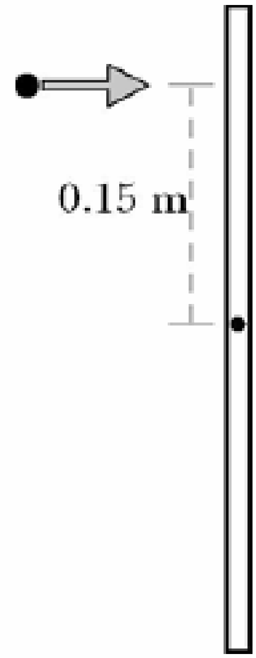
$$\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 = mg(y - y_0),$$

so

$$\omega = \sqrt{\omega_0^2 + \frac{2mg(y - y_0)}{I}}.$$

Plugging in numbers,

$$\omega = \sqrt{(12\ \mathrm{rad/s})^2 + \frac{2 \times 1.2\ \mathrm{kg} \times 9.8\ \mathrm{m/s^2} \times (0.66\ \mathrm{m} - 0.25\ \mathrm{m})}{0.06\ \mathrm{kg \cdot m^2}}} = \boxed{17.46\ \mathrm{rad/s}}.$$



Problem 15. A lump of clay of mass 0.07 kg traveling at speed 35 m/s strikes and sticks to a long thin rod of mass 1.2 kg and length 0.4 m as shown, a distance 0.15 m from the pivot. The rod was at rest before the clay hit it. The temperature of the rod and clay rises. The rod is free to turn on a nearly frictionless pivot at its center. Note that the moment of inertia about the center of a long thin rod of mass M and length L is $ML^2/12$.

(a) What is the angular speed of the rod just after impact? Start from a fundamental principle.

We will use the Angular Momentum Principle, $\vec{L}_f = \vec{L}_i + \vec{\tau}_{\text{net}}\Delta t$, picking as our system the rod and clay, but not the pivot or Earth, and we will measure angular momentum and torque from the pivot point. The system experiences a force at the pivot point when the clay hits the rod, which changes the angular momentum. However, the torque due to this force is zero, since we have $\vec{\tau} = \vec{r} \times \vec{F} = \vec{0} \times \vec{F} = 0$. We therefore have

$$\vec{L}_f = \vec{L}_i .$$

The initial angular momentum is

$$\vec{L}_i = \vec{r} \times \vec{p} = 0.15 \text{ m} \cdot 0.07 \text{ kg} \cdot 35 \text{ m/s} \otimes = 0.3675 \text{ kg} \cdot \text{m/s} \ \otimes .$$

The final angular momentum is

$$\vec{L}_f = \left(I_{\text{clay}} + I_{\text{rod}}\right)\vec{\omega} = \left(mr_\perp^2 + \frac{1}{12}ML^2\right)\vec{\omega} = \left(0.07 \text{ kg}\left(0.15 \text{ m}\right)^2 + \frac{1}{12}1.2 \text{ kg}\left(0.4 \text{ m}\right)^2\right)\vec{\omega}$$

$$= 1.7575 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \ \vec{\omega}.$$

Therefore,

$$\vec{\omega} = \frac{0.3675 \text{ kg} \cdot \text{m/s}}{1.7575 \times 10^{-2} \text{ kg} \cdot \text{m}^2} \otimes = \boxed{20.91 \text{ rad/s} \ \otimes}.$$

(b) What is the total kinetic energy of the rod plus clay just after impact?

The total kinetic energy is

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rot}} = K_{\text{rot}},$$

since after the impact there is no translational kinetic energy. The total kinetic energy is therefore

$$K_{\text{tot}} = \frac{1}{2} I_{\text{tot}} \omega^2 = \frac{1}{2} 1.7575 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \left(20.91 \text{ rad/s}\right)^2 = \boxed{3.84 \text{ J}}.$$

Problem 16. The mass of a mole of lead atoms is 207 grams, and measurement of Young's modulus for lead shows that the interatomic bond can be modeled as a spring with stiffness 5 N/m. Consider a nanoparticle consisting of 15 lead atoms.

(a) What is the entropy when this nanoparticle contains 12 quanta of energy?

We have $S = k\ln\Omega$, where $\Omega = \dfrac{(q+N-1)!}{q!(N-1)!}$. In this case, $q = 12$ and $N = 3\times 15 = 45$. Therefore,

$\ln\Omega = \ln\left(\dfrac{56!}{12!\,44!}\right) = \ln\left(5.58\times 10^{11}\right) = 27.05$. So $S = 1.38\times 10^{-23}\text{ J/K}\times 27.05 = \boxed{3.73\times 10^{-22}\text{ J/K}}$.

(b) What is the entropy when this nanoparticle contains 13 quanta of energy?

Now we have $q = 13$, so $\ln\Omega = \ln\left(\dfrac{57!}{13!\,44!}\right) = \ln\left(2.448\times 10^{12}\right) = 28.53$, and

$S = 1.38\times 10^{-23}\text{ J/K}\times 28.53 = \boxed{3.94\times 10^{-22}\text{ J/K}}$.

(c) What is the entropy when this nanoparticle contains 14 quanta of energy?

This time we have $q = 14$, so $\ln\Omega = \ln\left(\dfrac{58!}{14!\,44!}\right) = \ln\left(1.014\times 10^{13}\right) = 29.95$,

and $S = 1.38\times 10^{-23}\text{ J/K}\times 29.95 = \boxed{4.13\times 10^{-22}\text{ J/K}}$.

(d) What is the temperature of this nanoparticle when it contains 13 quanta of energy?

We have $\dfrac{\Delta S}{\Delta E} = \dfrac{1}{T} \Rightarrow T = \dfrac{\Delta E}{\Delta S}$. The change in energy from $q = 12$ to $q = 14$ is $2\hbar\omega$. We need

$$\omega = \sqrt{k_s/m} = \sqrt{5\text{ N/m}/\left(0.207\text{ g/mol}/6\times 10^{23}\text{ mol}^{-1}\right)} = 3.81\times 10^{12}\text{ s}^{-1}.$$

So

$T = 2\hbar\omega/\Delta S = 2\times 1.05\times 10^{-34}\text{ J/s}\times 3.81\times 10^{12}\text{ s}^{-1}/\left(4.13\times 10^{-22}\text{ J/K} - 3.73\times 10^{-22}\text{ J/K}\right) = \boxed{20\text{ K}}$.

(e) What is the heat capacity on a per atom basis when the nanoparticle contains 13 quanta of energy? (Check whether your result is reasonable, based on what you know about the high-temperature limit.)

We need the temp at 12.5 quanta and 13.5 quanta. $\Delta E = \hbar\omega = 4.0\times 10^{-22}\text{J}$. This gives

$T_{12.5} = 4.0\times 10^{-22}\text{ J}/\left(0.21\times 10^{-22}\text{ J/K}\right) = 19.0\text{ K}$ and $T_{13.5} = 4.0\times 10^{-22}\text{ J}/\left(0.19\times 10^{-22}\text{ J/K}\right) = 21.1\text{ K}$.

So $C = \Delta E/\Delta T = 4.0\times 10^{-22}\text{J}/\left(21.1\text{ K} - 19.0\text{ K}\right) = 1.90\times 10^{-22}$ J/K. Since there are 15 atoms, on a per atom basis, we have $C_{\text{per atom}} = 1.90\times 10^{-22}\text{ J/K}/15 = \boxed{1.27\times 10^{-23}\text{ J/K}}$.

The high temperature limit would be $C_{\text{high temp, per atom}} = 3k = 3\times 1.38\times 10^{-23}\text{ J/K} = 4.14\times 10^{-23}\text{ J/K}$.
The above is less than, but on the order of, the high temperature limit, so the result is reasonable.

Fundamental Concepts

What you must memorize:
(1) The Momentum Principle and the definition of momentum
(2) The relationship among position, velocity and time
(3) The Energy Principle, and the definition of work
(4) The Angular Momentum Principle

MULTIPARTICLE SYSTEMS

$K_{\text{total}} = K_{\text{trans}} + K_{\text{rel to CM}} = K_{\text{trans}} + K_{\text{rot}} + K_{\text{vib}}$ $\quad K_{\text{trans}} = \frac{1}{2} M_{\text{total}} v_{\text{cm}}^2 = \frac{p_{\text{total}}^2}{2M_{\text{total}}}$ nonrelativistically

$\vec{P}_{\text{total}} = M_{\text{total}} \vec{v}_{\text{cm}}$ nonrelativistically $\quad K_{\text{rot}} = \frac{L_{\text{rot}}^2}{2I} = \frac{1}{2} I \omega^2 \quad I = m_1 r_{\perp 1}^2 + m_2 r_{\perp 2}^2 + m_3 r_{\perp 3}^2 + \cdots$

$\Delta \vec{L}_{\text{tot}, A} = \vec{\tau}_{\text{net, ext}, A} \Delta t \quad \vec{L}_A = \vec{r}_A \times \vec{p}$ for a point particle $\quad \vec{\tau}_A = \vec{r}_A \times \vec{F}$

$\vec{L}_A = \vec{L}_{\text{trans}, A} + \vec{L}_{\text{rot}} = \left(\vec{R}_{\text{CM}, A} \times \vec{P}_{\text{total}} \right) + \vec{L}_{\text{rot}}$ for a multiparticle system $\quad \vec{L}_{\text{rot}} = I \vec{\omega}$

EVALUATING SPECIFIC PHYSICAL QUANTITIES

$\vec{F}_{\text{grav on 2 by 1}} = -G \frac{m_1 m_2}{|\vec{r}|^2} \hat{r}$ $\quad$ Near Earth's surface $\left| \vec{F}_{\text{grav}} \right| \approx mg \quad U_{\text{grav}} = -G \frac{m_1 m_2}{|\vec{r}|}$

$\vec{F}_{\text{elec on 2 by 1}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}|^2} \hat{r} \quad U_{\text{elec}} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}|} \quad \left| \vec{F}_{\text{spring}} \right| = k_s |s|$, opposite the stretch $\quad U_s = \frac{1}{2} k_s s^2$

$x = A \cos(\omega t)$ (solution to an idealized spring mass system) $\quad \omega = \sqrt{\frac{k_s}{m}} \quad \omega = \frac{2\pi}{T} = 2\pi f \quad Y = \frac{F_T / A}{\Delta L / L}$

$E^2 - (pc)^2 = \left(mc^2 \right)^2$; $\quad E = pc$ for massless photon

$\left(\frac{d|\vec{p}|}{dt} \right) \hat{p} = \vec{F}_{\text{parallel}} \quad |\vec{p}| \frac{d\hat{p}}{dt} = |\vec{p}| \frac{|\vec{v}|}{R} \hat{n} = \vec{F}_{\perp} \quad |\vec{v}| = \frac{2\pi |\vec{r}|}{T}$

Macro/micro connections: macro measurement of density = $\frac{m}{d^3}$ (micro quantities)

macro measurement of Young's modulus $Y = \frac{k_{s,i}}{d}$ (micro quantities)

macro measurement of speed of sound $v = d \sqrt{\frac{k_{s,i}}{m_a}}$ (micro quantities)

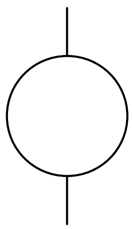
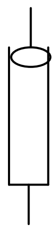
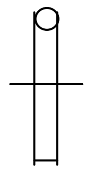
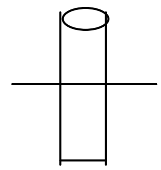
$\Delta E_{\text{thermal}} = mC\Delta T$, where m is in grams if C is in (J/K)/gram $\quad$ Power = $\frac{\text{energy}}{\text{time}}$ (watts = J/s)

$E_N = N \left(\hbar \sqrt{\frac{k_s}{m}} \right)$; $\quad$ hydrogen atom: $E_N = -\frac{13.6 \text{ eV}}{N^2} \quad (N = 1, 2, \ldots)$

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!} \qquad S = k\ln\Omega \qquad \frac{1}{T} = \frac{dS}{dE}$$

$C = \dfrac{\Delta E}{\Delta T}$ (high-temperature limit: $\dfrac{1}{2}k$ per quadratic energy term; solid is $3k$/atom)

$$\vec{A}\times\vec{B} = \left\langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \right\rangle \qquad \left|\vec{A}\times\vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right|\sin\theta = A_\perp B = AB_\perp$$

Sphere	Cylinder or disk	Thin rod about axis shown	Solid cylinder about axis shown
			
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Physical Constants

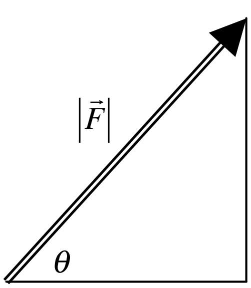
$G = 6.7\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$ $\qquad g = 9.8$ N/kg $\qquad \dfrac{1}{4\pi\varepsilon_0} = 9\times10^{9}\ \mathrm{N\cdot m^2/C^2}$

$c = 3\times10^{8}$ m/s $\qquad h = 6.6\times10^{-34}\ \mathrm{J\cdot s}$ $\qquad \hbar = 1.05\times10^{-34}\ \mathrm{J\cdot s}$

$M_{\mathrm{Earth}} = 6\times10^{24}$ kg $\qquad M_{\mathrm{Moon}} = 7\times10^{22}$ kg $\qquad M_{\mathrm{Sun}} = 2\times10^{30}$ kg

Radius of the Earth = 6.4×10^{6} m $\qquad$ Radius of the Moon = 1.75×10^{6} m
Distance from Earth to Moon = 4×10^{8} m $\qquad$ Distance from Sun to Earth = 1.5×10^{11} m
Avogadro's number = 6×10^{23} molecules/mole $\qquad$ Typical atomic radius $r\approx10^{-10}$ m
$m_{\mathrm{electron}} = 9\times10^{-31}$ kg $\qquad m_{\mathrm{proton}} \approx m_{\mathrm{neutron}} \approx m_{\mathrm{hydrogen\ atom}} = 1.7\times10^{-27}$ kg
$e = 1.6\times10^{-19}$ C where e = charge on proton = -charge on electron $\qquad$ 1 eV = 1.6×10^{-19} J
Heat capacity of water = 4.2 (J/K)/gram

Trigonometric properties



$F_y = \left|\vec{F}\right|\sin\theta$

$F_x = \left|\vec{F}\right|\cos\theta$

$$\cos\theta = \frac{\mathrm{adj}}{\mathrm{hyp}} = \frac{F_x}{\left|\vec{F}\right|} \qquad \sin\theta = \frac{\mathrm{opp}}{\mathrm{hyp}} = \frac{F_y}{\left|\vec{F}\right|}$$