Macroscopic and Microscopic Springs, Part 1

GOALS

In this lab you will investigate the properties of different spring-like systems. The questions you will need to answer are:

1) **In what way is a straight metal wire like a spring?**
2) **What is the "stretchiness" of copper, independent of size & shape of an object?**
3) **How can one relate the macroscopic "stretchiness" of a material to the microscopic stiffness of the interatomic bonds?**

You will record and analyze your data in Excel. You should download the Excel spreadsheet from the course website (see Lab Calendar). You will submitt via WebAssign the Excel document along with a lab report. The guidelines for this lab report can be found on the course website.

Note: An example of EVERY calculation (even those done in Excel) should be in your report.

BACKGROUND

1) Spring forces

Remember that the magnitude of the force exerted by a spring on an object attached to it is linearly proportional to the absolute value of the stretch of the spring. The stretch s can be positive or negative, and is defined as the difference between the current length of the spring L and its original, relaxed length L_0. Sometimes the symbol ΔL is used to represent the stretch: $s = \Delta L = L - L_0$. The spring constant k_s represents the stiffness of a particular spring, and has units of N/m.

$$|F| = k_s\,|s| = k_s\,|\Delta L|$$

2) Young's modulus

The stiffness of a wire depends on the length of the wire and the thickness of the wire, so different wires made from the same metal will have different stiffnesses. It is useful to have a measure of the stiffness of a particular material (such as aluminum, copper, gold, carbon nanotubes) independent of the dimensions of the wire. Calculating Young's modulus is a way to measure the "springiness" of a material and factor out the size and shape of the particular wire

Young's modulus is the ratio of:

Force exerted per square meter of cross-sectional area (F/A) ("stress")

to

Fractional stretch of the wire ($\Delta L/L_0$) ("strain")

$$Y = \frac{(F/A)}{(\Delta L/L_0)}$$

1) Interatomic spring stiffness

Remember that our simple model of a solid object is a bunch of tiny balls (atoms) that are held together by springs (chemical bonds). The relaxed length of the little spring between two atoms (the interatomic bond) is just the distance from the center of one atom to the center of the other atom, d. For our model, this is just twice the radius of the one of the atoms, since the electron cloud of the atom fills in the extra space. A simple cubic arrangement of the atoms would mean that the volume that each atom fills would be $d \times d \times d$ and would have a cross-sectional area of $d \times d$.

Since the interatomic bonds are modeled as springs, they also have a stiffness $k_{s,I}$ that relates the interatomic force to how much those interatomic bonds stretch.

$$|F| = k_{s,i}\,|s| = k_{s,i}\,|\Delta L|$$

STRAIGHT METAL WIRE

1) Is a solid metal straight wire like a spring? If so, in what way?
2) If you consider the wire to be a spring, what is its spring stiffness?
3) Express the spring-like properties of the particular metal in a way that is independent of the physical dimensions of the wire (Young's modulus).
4) Relate the macroscopic spring-like properties of the particular metal to the stiffness of that metal's interatomic bonds (interatomic spring stiffness).

BEFORE STARTING, READ THE NOTES ON EXPERIMENTAL PROCEDURE (Sec 1-4)

Apparatus: straight metal wire stretched on a frame, with a hanger for weights and a 0.001 inch scale for determining the position of the end of the wire, caliper.

Procedural Overview:

- To determine if a straight wire behaves like a spring, measure the following:
  - Length of the wire (without any weights on it)
  - Position of the end of the wire as you add different amounts of weight (0.1 – 0.9 kg)
  - Diameter of the wire
- Record data in a table
  - To provide a measure of how reproducible your measurements are, calculate and record in your table the variation of your data (see section 4 below).
- Plot a graph of applied force vs. stretch
- Use data to calculate Young's Modulus
- Use data to approximate the interatomic spring stiffness

Notes on Experimental Procedure (*READ BEFORE STARTING*):

1) If the wire has kinks in it, you will measure the springiness of the "coiled" wire instead of the stiffness of a straight wire. *Remove the kinks before starting your measurements.*
   - Add 1 kg to the platform hanging from your wire. Remove the 1 kg. Repeat. This helps get small kinks out of the wire, so the "straight" wire doesn't act like a coiled spring.
2) Look at the scale on the apparatus, and make sure you know what the divisions are, and how to read the scale. You should be able to measure to the nearest 0.001 in.
3) You need to make ***three*** sets of measurements, so you can get average values.
   - Take measurements with 100 g, then 200 g, 300 g, and so on up to 900 g
   - Then take all the masses off and start from zero again.
   - ***If your sets of measurements differ by more than 0.002 in, repeat the measurements***
   - Discard earlier measurements since this was probably getting more kinks out of the wire.
4) The variation in the measurements (the "reproducibility") gives you an indication of how meaningful the individual measurements are. Record your estimate of the variation.
   - Suppose you had measurements that are 5.5, 5.7, 5.4, 5.6, 5.8, and 5.6 mm. The average is (5.5+5.7+5.4+5.6+5.8+5.6)/6 = 5.6, with a range from a minimum of 5.4 to a maximum of 5.8. A compact way to report this average with the approximate variation is 5.6± 0.2 mm.

NOTE: If your variation is larger than about 0.002 mm, you should repeat your measurements.

Verify that you have the following measurements:

- ***Three*** values for the ***position*** of the end of the wire with each of the masses
- The total ***length*** of the wire and its ***diameter***

CHECKPOINT Check how reasonable your data is by comparing it with another group

If you have not yet done parts 2 and 3, do those now!

If you do not complete the analysis in class, you must meet with your group outside of class.

Notes on Data Analysis **(*READ BEFORE STARTING*):**

Analysis of straight wire data

1) Graphing your data
   - Enter your data into Excel, and use it to make a graph of the force exerted on the wire (on the y-axis) versus the stretch of the wire (on the x-axis).
     - ***Note: you will have to compute both of these quantities from your measured data.*** You can do it by hand or create a formula in Excel to do it.
     - Do not include your 0-point since this may be the least accurate due to remaining kinks.
   - Fit your data to a trendline (***Note: remove the gray background on the plot***)
   - From your data, determine the spring stiffness of the wire.
2) Young's modulus
   - You can use your value for the spring constant of your wire to determine Young's modulus: At equilibrium $F = k_s s$. Also, $\Delta L = s$, so

$$Y = \frac{(F/A)}{(\Delta L/L_0)} = \frac{(k_s s/A)}{(s/L_0)} = \frac{k_s L_0}{A}$$

   - What is the cross-sectional area of your wire?
   - Determine Young's modulus for the material your wire is made of (brass).
   - Record your calculations in your notes and include them in your lab report.
   - ***Address the following questions in your lab report:***
     - If the cross-sectional area of the wire were doubled, would the wire stretch more, less, or the same amount when you hung a 800 g mass on it? Why?
       - How does this compare to placing two springs in parallel?
     - If the wire were twice as long, would it stretch more, less, or the same amount when you hung a 800 g mass on it? Why?
       - How does this compare to placing two springs in series?
     - The quantity *F/A* is called "stress". What are its units?
     - The fractional stretch $\Delta L/L_0$ is called "strain". What are its units?

Note: For comparison, Young's modulus for aluminum is 6.2e10 N/m^2, for lead it is 1.6e10 N/m^2.

3) Interatomic Spring Stiffness
   - You can use your value for Young's modulus to approximate the interatomic stiffness: Using our expression for Y and the fact that the cross-sectional area of the bond is $A \approx d^2$ and the relaxed length is $L_0 = d$:

$$Y = \frac{k_s L_0}{A} = \frac{k_{s,i} d}{d^2} = \frac{k_{s,i}}{d}$$

   - What is the length of the interatomic bond?
   - Determine the interatomic spring stiffness of copper.

Lab Report

Work with your partners on a lab report that follows the guidelines found on the course website. ***Note: Only one report should be submitted by the Recorder to WebAssign in .doc or .pdf format. If you have problems with file size, pdf files are usually smaller.***