Formula sheet

Unit vector $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$ Displacement $\vec{r_f} - \vec{r_i} = \Delta \vec{r}$ Velocity $\vec{v} = \frac{\vec{r_f} - \vec{r_i}}{t_f - t_i} = \frac{\Delta \vec{r}}{\Delta t}$ For motion along a curved path,

$$\left|\frac{d\vec{p}}{dt}\right|\hat{p} = \vec{F}_{\parallel} \text{ and } |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \vec{F}_{\perp} \text{ or } p\frac{v}{R} = F_{\perp}$$

where R is the radius of the "kissing circle" and \hat{n} points in the direction of \vec{F}_{\perp} (toward center of circle).

Speed of light
$$c = 3 \times 10^8 \text{m/s}$$
 $G = 6.7 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$ $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$ $0^\circ \text{C} = 273 \text{ K}$

Magnitude of gravitational force $\left|\vec{F}_{\rm gr}\right| = G \frac{Mm}{|\vec{r}|^2}$ Gravitational potential energy $U_{\rm gr} = -\frac{GMm}{|\vec{r}|}$ Near the Earth's surface $\left|\vec{F}_{\rm gr}\right| \approx mg$ and $\Delta U_{\rm gr} \approx \Delta(mgy)$ $g = +9.8 \frac{N}{\rm kg}$

Magnitude of electric force
$$\left|\vec{F}_{\rm el}\right| = \frac{1}{4\pi\epsilon_0} \frac{|Qq|}{|\vec{r}|^2}$$
 Electric potential energy $U_{\rm el} = \frac{1}{4\pi\epsilon_0} \frac{Qq}{|\vec{r}|}$

Magnitude of spring force $\left|\vec{F}_{\rm spring}\right| = k_s |s|$ Spring potential energy $U_{\rm spring} = \frac{1}{2}k_s s^2 + U_0$ where a negative U_0 makes $U_{\rm spring} < 0$. It is the same value for initial and final states, so it cancels from the energy principle formula.

Idealized mass-spring oscillator
$$x = A \cos \omega t = A \cos \left(\sqrt{\frac{k_s}{m}} t \right), \quad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

 $Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L} = \frac{k_{s,i}}{d} \text{ (in terms of atomic quantities)} \qquad v_{\text{sound}} = d\sqrt{\frac{k_{s,i}}{m}}$
Kinetic energy, valid at any speed less than $c \quad K = E - mc^2$
Kinetic energy, valid at speeds much less than $c \quad K \approx \frac{1}{2}m|\vec{v}|^2$
Relationship between relativistic energy and momentum of a particle $E^2 - (pc)^2 = (mc^2)^2$

More equations on next page

$$\vec{L}_A = \langle (yp_z - zp_y), (zp_x - xp_z), (xp_y - yp_x) \rangle \qquad \vec{\tau}_A = \vec{r}_A \times \vec{F} \qquad K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{L_{\text{rot}}^2}{2I}$$
$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots$$

ways to arrange q quanta of energy among N one-dimensional oscillators

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!}$$

Entropy $S=k\ln\Omega$ where $k=1.38\times 10^{-23}~{\rm J/K}$

Temperature : $\frac{1}{T} = \frac{\Delta S}{\Delta E}$ Specific heat capacity *per atom* : $C = \frac{\Delta E_{\text{atom}}}{\Delta T}$

Probability of finding energy E in small system in contact with large reservoir is proportional to $\Omega(E)e^{-E/kT}$