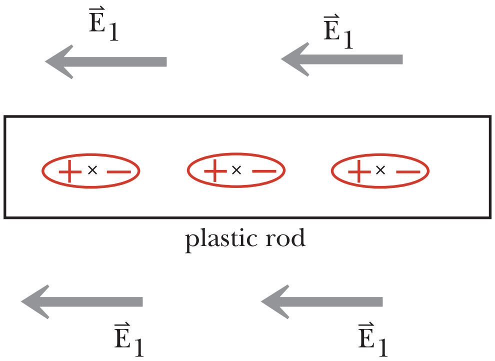
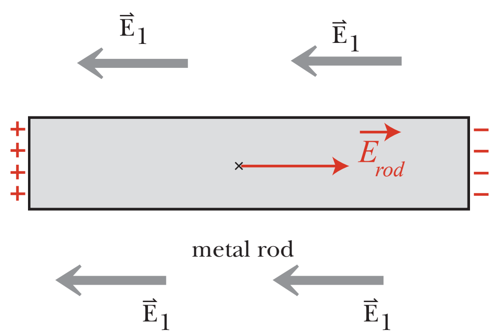
1. (12 pts) In a region of space there is an electric field $\vec{E}_1$ that is uniform in magnitude and direction, due to charges that are not shown. The magnitude and direction of $\vec{E}_1$ are indicated by arrows shown in the diagrams below.

   (a) (3 pts) A plastic rod is placed in this region. Show the polarization of a molecule inside the plastic rod at each of the three locations indicated by an x. If a molecule at a given location would not be polarized, state this explicitly.



   (b) (3 pts) Now the plastic rod is removed and replaced by a metal rod. On the diagram below, show the charge distribution in and/or on the metal rod.

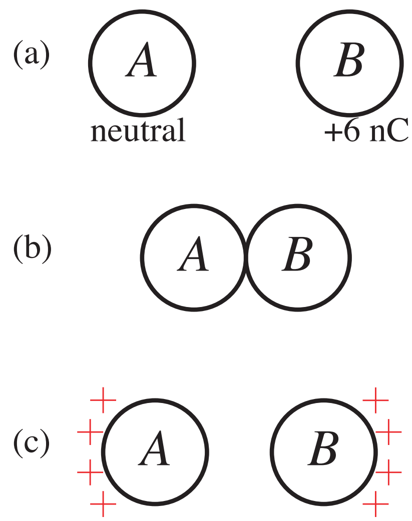


   (c) (3 pts) In the center of the metal rod, at the location marked by an x, draw an arrow representing the electric field at that location due only to the charges in and/or on the metal rod. Label the arrow $\vec{E}_{rod}$. Or, if this field has zero magnitude, state this explicitly.

   (d) (3 pts) How does this magnitude of $\vec{E}_{rod}$ compare to the magnitude of $\vec{E}_1$?

   The magnitudes are equal: $|\vec{E}_{rod}| = |\vec{E}_1|$.

2. (8 pts) A and B are identical metal spheres. Initially sphere A is neutral and the net charge of sphere B is +6 nC (+6e-9 C), and the two sphere are separated by 3 cm (fig. a). Next, the two spheres are brought into contact (fig. b). The spheres are then moved apart 3 cm (fig. c).



(a) (4 pts) Draw the approximate charge distributions on both spheres in the final configuration (fig. c).

(b) (4 pts) What is the final charge of sphere A? Give both the magnitude and sign of the charge.

The will have the same amount of charge, so it is +6 nC/2 = $\boxed{+3\ \text{nC}}$.

3. (6 pts) True or False

__T__ The net charge of a dipole is zero

__T__ The electric field at any location in space, due to a dipole, is the vector sum of the electric field due to the positive charge and the electric field due to the negative charge

__F__ At a small distance d from the dipole, where $d \ll s$ (the separation between the charges), the magnitude of the electric field due to the dipole is proportional to $1/d^3$

__F__ The net electric field due to a dipole is zero, since the contribution of the negative charge cancels out the contribution due to the positive charge

__T__ A dipole consists of two particles whose charges are equal in magnitude but opposite in sign

__T__ At a large distance d from the dipole, where $d \gg s$ (the separation between the charges), the magnitude of the electric field due to the dipole is proportional to $1/d^3$

4. (20 pts) A carbon nucleus (6 protons and 6 neutrons, with mass = 2×10^{-26} kg) accelerates to the left due to electric forces, and the initial magnitude of the acceleration is 5×10^{13} m/s^2.

(a) (5 pts) What is the direction of the electric field that acts on the carbon nucleus?

The force on the electron is $\vec{F} = m\vec{a}$, which is equal to $q\vec{E}$. Since $q > 0$, the electric field is in the same direction as the the force (and the acceleration) so $\boxed{\text{the electric field is to the left}}$.

(b) (5 pts) What is the magnitude of the electric field that acts on the carbon nucleus?

The force on the electron is $F = ma$, which is equal to qE, so

$$E = \frac{ma}{q} = \frac{2 \times 10^{-26}\ \text{kg} \times 5 \times 10^{13}\ \text{m/s}^2}{6 \times 1.6 \times 10^{-19}\ \text{C}} = \boxed{1.04\times10^{6}\ \text{N/C}}.$$

(c) (10 pts) If this acceleration is due solely to a single lithium nucleus (3 protons and 4 neutrons), where is the lithium nucleus initially located?

The lithium has a positive charge, so the nucleus should be to the right of the carbon nucleus. We have

$$E = \frac{1}{4\pi\epsilon_0}\frac{q_{Li}}{r^2},$$

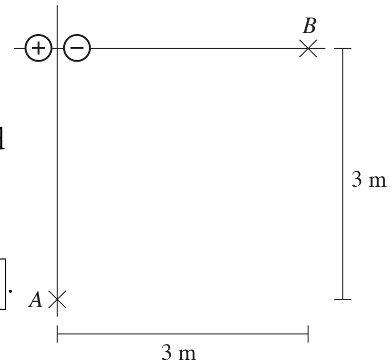
or

$$\begin{aligned} r &= \sqrt{\frac{q_{Li}}{4\pi\epsilon_0 E}} \\ &= \sqrt{\frac{3 \times 1.6 \times 10^{-19}\ \text{C} \times 9 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}{1.04 \times 10^{6}\ \text{N/C}}} \\ &= \boxed{6.45 \times 10^{-8}\ \text{m to the right of the carbon nucleus}}. \end{aligned}$$

5. (14 pts) The diagram shows a dipole centered about the origin.

Note that the diagram is not to scale and the dipole separation is much smaller than 1 meter.

(a) (4 pts) If the magnitude of the electric field at location A is 3.8×10^4 N/C, what is the magnitude of the electric field at location B?



There is a factor of 2 difference between the magnitude of the electric field on the axis compared to on the perpendicular axis. We therefore have

$$|\vec{E}_B| = 2|\vec{E}_A| = 2 \times 3.8 \times 10^4 \text{ N/C} = \boxed{7.6 \times 10^4 \text{ N/C}}.$$

(b) (10 pts) Calculate the dipole moment of the dipole. Show all your work.

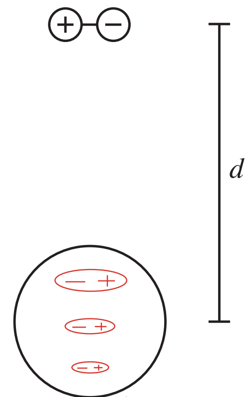
We have

$$\begin{aligned} |\vec{E}_A| &= \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \\ \Longrightarrow \quad p &= \frac{r^3 |\vec{E}_A|}{1/4\pi\epsilon_0} \\ &= \frac{(3 \text{ m})^3 \times 3.8 \times 10^4 \text{ N/C}}{9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} \\ &= \boxed{11.4 \times 10^{-5} \text{ C} \cdot \text{m}}. \end{aligned}$$

6. (20 pts) **Note that the diagram is not to scale; the dipole is shown larger than the actual size.**

   A permanent dipole is at a distance d from the center of a neutral spherical insulator.

   (a) (3 pts) On the diagram draw the approximate charge distribution *in and/or on the insulator.*



Now the insulator is removed and replaced by a neutral metal sphere.

The relative lengths of the arrows in your diagram below must be correct. Label your arrows clearly. Arrows without labels will be counted as wrong.

If a quantity is zero, you must state that explicitly.

The diagram is on the next page.

   (b) (3 pts) At the center of the metal sphere, draw an arrow to represent the electric field due to just the dipole and label it $\vec{E}_d$.

   (c) (3 pts) Draw the charge distribution *in and/or on the metal sphere.*

   (d) (3 pts) At the center of the metal sphere, draw an arrow to represent the electric field due to the charges in and/or on the sphere and label it $\vec{E}_{pol}$.

   (e) (3 pts) at the center of the sphere, draw an arrow to represent the net electric field and label it $\vec{E}_{net}$.

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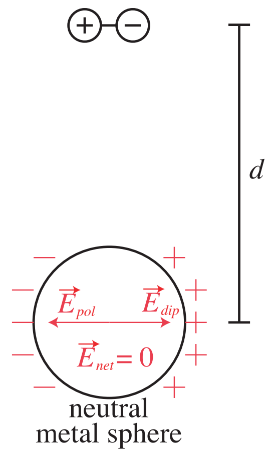
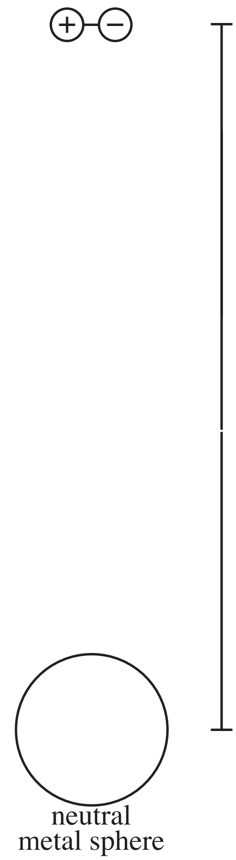


The metal sphere is now moved away so that the distance to the center of the sphere is $3d$.

(f) (5 pts) Calculate the ratio $|\vec{E}_{pol,3d}|/|\vec{E}_{pol,d}|$.

Inside the conductor, we have

$$\vec{E}_{net} = 0 = \vec{E}_{pol} + \vec{E}_{dip}$$



Therefore,

$$|\vec{E}_{pol}| = |\vec{E}_{dip}|$$

We therefore have

$$|\vec{E}_{pol,d}| = |\vec{E}_{dip,d}| = \frac{1}{4\pi\epsilon_0}\frac{p}{d^3},$$

and

$$|\vec{E}_{pol,3d}| = |\vec{E}_{dip,3d}| = \frac{1}{4\pi\epsilon_0}\frac{p}{(3d)^3}.$$

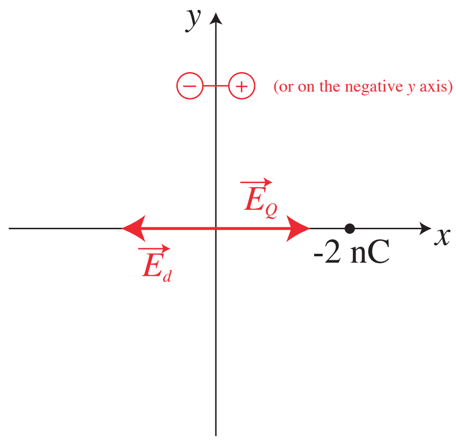
Taking the ratio,

$$\frac{|\vec{E}_{pol,3d}|}{|\vec{E}_{pol,d}|} = \frac{1/(4\pi\epsilon_0)\ p/(3d)^3}{1/(4\pi\epsilon_0)\ p/d^3} = \frac{d^3}{27d^3} = \frac{1}{27}.$$

7. (20 pts) A hollow glass ball with radius 2 cm has a charge of -4 nC spread uniformly over its surface. The center of the ball is located at $\langle 12, 0, 0\rangle$ cm.

Note that the diagram is not to scale.

(a) (2pts) On the diagram draw an arrow to represent the electric field vector at the origin due to the charged ball. Label it $\vec{E}_Q$.



(b) (3 pts) You are asked to place a dipole centered on the y-axis, with its center 5 cm from the origin. On the diagram, show how you would orient this dipole on the y-axis so that the net field at the origin is zero. Note that the dipole separation s is much smaller than 5 cm.

(c) (2 pts) On the diagram, draw an arrow to represent the electric field at the origin due to the dipole and label it $\vec{E}_d$. **The relative lengths of the two arrows representing $\vec{E}_Q$ and $\vec{E}_d$ must be correct. Arrows without labels will be counted as wrong.**

(d) (13 pts) If the dipole charges are +6 nC and -6 nC, determine the dipole separation. Show all your work.

We have, since the net field is equal to zero, $\vec{E}_Q = -\vec{E}_d$, or taking magnitudes,

$$|\vec{E}_Q| = \frac{1}{4\pi\epsilon_0}\frac{|Q|}{r_Q^2} = |\vec{E}_d| = \frac{1}{4\pi\epsilon_0}\frac{qs}{r_d^3} \quad\Longrightarrow\quad s = \frac{|Q|}{q}\frac{r_d^3}{r_Q^2},$$

where $|Q| = 4$ nC, $q = 6$ nC, $r_Q = 12$ cm, and $r_d = 5$ cm. Plugging in numbers,

$$s = \frac{4\text{ nC}}{6\text{ nC}}\frac{(5\text{ cm})^3}{(12\text{ cm})^2} = 0.58\text{ cm} = \boxed{5.8\text{ mm}}.$$

Fundamental Concepts

Equations you must know:

(1) Electric field of a point charge (2) Relationship between electric field and electric force

Other fundamental concepts: the Superposition Principle Conservation of charge

Specific Results

Electric field due to a uniformly charged spherical shell: outside shell, like point charge; inside shell, 0

$$\left|\vec{E}_{\text{dipole}}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3} \text{ along dipole axis, where } r \gg s$$

$$\left|\vec{E}_{\text{dipole}}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3} \text{ along perpendicular axis, where } r \gg s$$

electric dipole moment $p = qs$

Physical Constants

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \qquad \epsilon_0 = 9 \times 10^{-12} \frac{\mathrm{C^2}}{\mathrm{N \cdot m^2}} \qquad e = 1.6 \times 10^{-19}\ \mathrm{C}$$

$\mathrm{c} = 3\times10^8$ m/s $m_{\text{proton}} \approx m_{\text{neutron}} \approx m_{\text{hydrogen atom}} = 1.7\times10^{-27}$ kg $m_{\text{electron}} = 9\times10^{-31}$ kg

$g = 9.8$ N/kg 6.02×10^{23} molecules/mole atomic radius $\approx 10^{-10}$ m proton radius $\approx 10^{-15}$ m

Geometry

area of circle $= \pi r^2$ circumference of a circle $= 2\pi r$

area of curved surface of cylinder $= 2\pi r L$ surface area of sphere $= 4\pi r^2$

volume of sphere $= \frac{4}{3}\pi r^3$