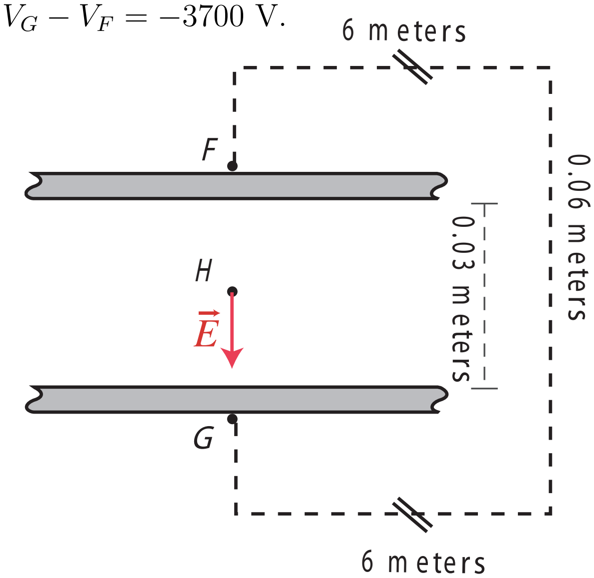
1. (6 pts) Locations F and G are just outside two uniformly charged large metal plates, which are 3 cm apart. Measured along the path indicated by the dotted line, the potential difference $V_G - V_F = -3700$ V.



(a) (2 pts) Draw an arrow indicating the direction of the electric field at location H, midway between the plates.

The direction going inside the capacitor is from F to G, or down. The potential is negative, which means that the electric field points in the same direction as the direction of the path.

(b) (4 pts) What is the magnitude of the electric field at location H? Show your work.

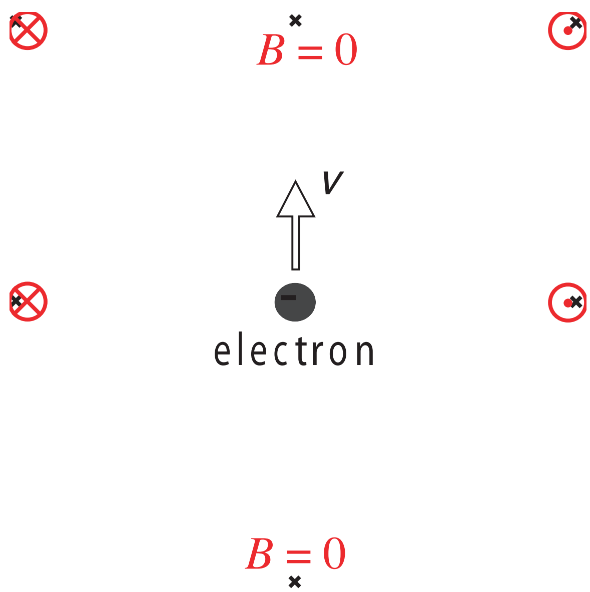
We have

$$\Delta V = V_G - V_F = -\int \vec{E} \cdot d\vec{l} = -|\vec{E}| \times\ 0.03\text{m}.$$

So

$$|\vec{E}| = \frac{3700 \text{ V}}{0.03 \text{ m}} = \boxed{1.23 \times 10^5 \text{ V/m}}.$$

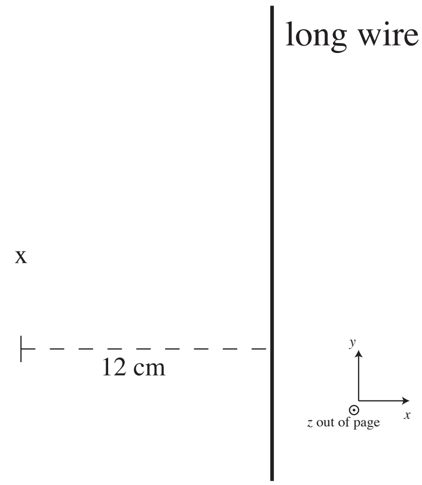
2. (6 pts) An electron is moving as shown. At the locations marked ×, show the direction of the magnetic field made by the electron. If the field is zero, say so. Use standard notation for out of the plane (⊙) and into the plane (⊗).



RHR: Circle fingers of right hand around velocity vector with your thumb pointing in the direction of the velocity. Your fingers go into the page on the right, out on the left. Since it is an electron, we need to flip the vectors, giving out on the right, in on the left.

3. (10 pts) A long straight wire of length 12 m is placed parallel to the y-axis. At a location 12 cm from the wire, marked "x" in the diagram), the magnetic field was measured to be into the page with a magnitude of 9.4e-6 T. **The wire is much longer than it is shown in the diagram.**

What is the magnitude and direction of the conventional current flowing in the long wire? *Show all steps in your work.*



RHR: Circle fingers of right hand around wire so that they go into the page at "x". Your thumb will be pointing up $\boxed{\text{in the -y direction}}$.

Since the length of the wire is much longer than the distance from the wire, we can use the approximate formula for an infinitely long wire:

$$|\vec{B}_{wire}| = \frac{\mu_0}{4\pi}\frac{2I}{r} \quad (r \ll L).$$

Therefore,

$$I = \frac{r|\vec{B}_{wire}|}{2\mu_0/4\pi}.$$

Plugging in numbers

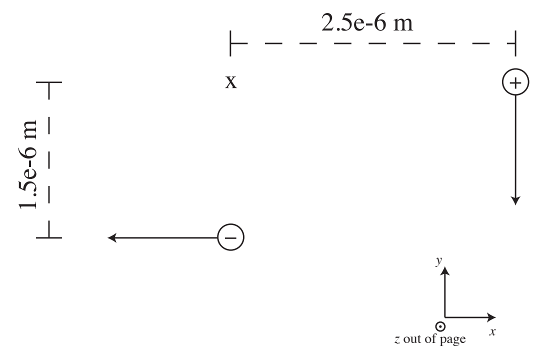
$$\begin{aligned} I &= \frac{0.12 \text{ m} \times 9.4 \times 10^{-6} \text{ T}}{2 \times 10^{-7} \text{ T} \cdot \text{m/A}} \\ &= \boxed{5.64 \text{ A}}. \end{aligned}$$

4. (20 pts) An electron and an alpha particle ($q = 2e$) are moving with equal speeds of 3.2×10^6 m/s in the directions shown.

   (a) (2 pts) What is the direction of the magnetic field due to the electron at the location marked x? *Circle one:*

   +x   -x   +y   -y   [+z]   -z   Something else

   (b) (5 pts) Calculate the magnitude of the magneteic field $\vec{B}_1$ due to just the electron at the location marked x. *Show all steps in your work.*



We have

$$\begin{aligned} B_e &= \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} \\ &= 10^{-7}\frac{\mathrm{T\cdot m}}{\mathrm{A}}\frac{1.6\times10^{-19}\mathrm{C}\times 3.2\times 10^6\ \mathrm{m/s}\times\sin(90^\circ)}{(1.5\times10^{-6}\ \mathrm{m})^2} = \boxed{2.28\times10^{-8}\ \mathrm{T}}. \end{aligned}$$

   (c) (2 pts) What is the direction of the magnetic field due to the alpha particle at the location marked x? *Circle one:*

   +x   -x   +y   -y   +z   [-z]   Something else

   (d) (5 pts) Calculate the magnitude of the magnetic field $\vec{B}_2$ due to just the alpha particle at the location marked x. *Show all steps in your work.*

Now we have We have

$$\begin{aligned} B_\alpha &= \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2} \\ &= 10^{-7}\frac{\mathrm{T\cdot m}}{\mathrm{A}}\frac{2\times1.6\times10^{-19}\mathrm{C}\times 3.2\times 10^6\ \mathrm{m/s}\times\sin(90^\circ)}{(2.5\times10^{-6}\ \mathrm{m})^2} = \boxed{1.64\times10^{-8}\ \mathrm{T}}. \end{aligned}$$

Continued on next page

(e) (4 pts) Calculate the magnitude of the new magnetic field $\vec{B}_{net}$ due to the electron and the alpha particle at the location marked x. *Show all steps in your work.*

We have

$$\vec{B}_{\text{net}} = B_e\hat{z} + B_\alpha(-\hat{z}) = (B_e - B_\alpha)\hat{z},$$

and so

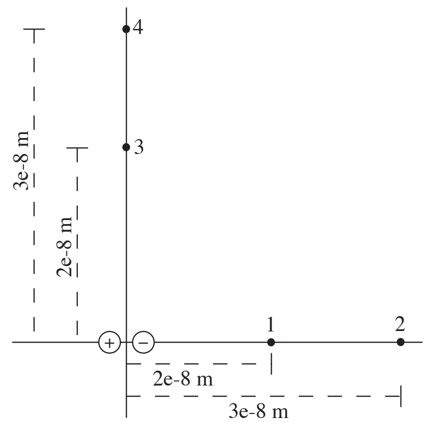
$$B_{\text{net}} = 2.28 \times 10^{-8}\ \text{T} - 1.64 \times 10^{-8}\ \text{T} = \boxed{6.4 \times 10^{-9}\ \text{T}}.$$

(f) (2 pts) What is the direction of the magnetic field due to the two moving charges at the location marked x? *Circle one:*

+x   -x   +y   -y $\boxed{\text{+z}}$ -z   Something else

5. (20 pts) A hydrogen chloride (HCl) molecule consists of a positive ion with charge $+e$ and a negative ion with charge -e, separated by a distance of 2×10^{-11} m, as shown in the diagram. Locations 1, 2, 3, and 4 are shown in the diagram. **Note that the diagram is not to scale.**

(a) (12 pts) Location 1 is 2×10^{-8} m from the center of the molecule, and location 2 is 3×10^{-8} m from the center of the molecule. Calculate the potential difference $V_2 - V_1$, both magnitude and sign. *Show all steps in your work.*



$$\begin{aligned} V_2 - V_1 &= -\int_1^2 \vec{E} \cdot d\vec{l} \\ &= -\int_1^2 (-E_x) dx \\ &= \frac{1}{4\pi\epsilon_0} 2qs \int_1^2 \frac{dx}{x^3} \\ &= \frac{1}{4\pi\epsilon_0} 2qs \left(-\frac{1}{2x^2}\right)_1^2 \\ &= \frac{1}{4\pi\epsilon_0} qs \left(-\frac{1}{x^2}\right)_1^2 \\ &= -9 \times 10^9 \cdot 1.6 \times 10^{-19} \cdot 2 \times 10^{-11} \left[\frac{1}{(3.8 \times 10^{-8})^2} - \frac{1}{(2 \times 10^{-8})^2}\right] \text{V} \\ &= \boxed{4.0 \times 10^{-5} \text{ V}}. \end{aligned}$$

(b) (8 pts) Location 3 is 2×10^{-8} m from the center of the molecule, and location 4 is 3×10^{-8} m from the center of the molecule. Calculate the potential difference $V_4 - V_3$, both magnitude and sign. *Show all steps in your work.*

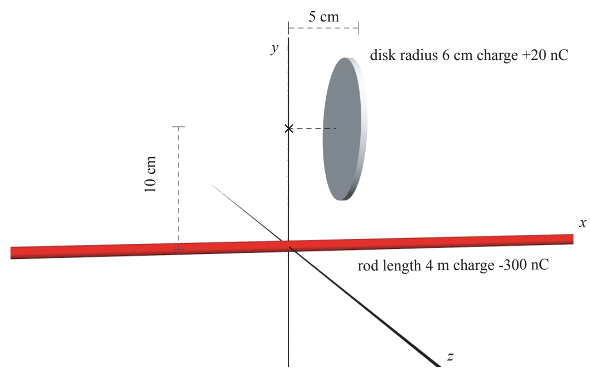
This time we have

$$V_4 - V_3 = -\int_3^4 \vec{E} \cdot d\vec{l} = \boxed{0},$$

since $\vec{E}$ is in the $\hat{x}$ direction, and $d\vec{l}$ is in the $\hat{y}$ direction.

6. (20 pts) A thin plastic rod, of length 4 m, lies along the x-axis (only a portion of the rod is shown in the diagram). It has a charge of -300 nC uniformly distributed over its surface. A thin disk, of radius 6 cm, is centered at $\langle 5, 10, 0\rangle$ cm. It has a uniform charge of +20 nC. **Note that the diagram is not to scale.**

Calculate the force on a small ball with charge -8 nC placed at $\langle 0, 10, 0\rangle$ cm. **Your answer must be a vector.** *Clearly show all steps in your work.*



The rod is negatively charged, giving a component of the electric field in the $-\hat{y}$ direction. The disk is positively charged, giving a component of the electric field in the $-\hat{x}$ direction. Since the rod is so long (compared to the distance from it that we are measuring $\vec{E}$), we can use the approximate formula:

$$\begin{aligned} E_{\text{rod}} &= \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r} \\ &= 9\times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\frac{2\cdot 300\times 10^{-9}\ \text{C}/4\ \text{m}}{0.10\ \text{m}} \\ &= 1.35\times 10^4\ \text{N/C}. \end{aligned}$$

For the disk we have

$$\begin{aligned} E_{\text{disk}} &= \frac{Q/A}{2\epsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right] \\ &= \frac{20\times 10^{-9}\text{C}/[\pi(0.06\ \text{m})^2]}{2\times 8.85\times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)}\left[1-\frac{0.05\ \text{m}}{\sqrt{(0.05\ \text{m})^2+(0.06\ \text{m})^2}}\right] \\ &= 3.6\times 10^4\ \text{N/C}. \end{aligned}$$

Therefore, the electric field is

$$\vec{E} = \langle -3.6\times 10^4, -1.35\times 10^4, 0\rangle\ \text{N/C}.$$

The force is then

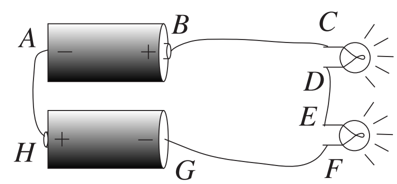
$$\begin{aligned} \vec{F} &= q\vec{E} = -8\times 10^{-9}\ \text{C}\times\langle -3.6\times 10^4, -1.35\times 10^4, 0\rangle\ \text{N/C} \\ &= \boxed{\langle 2.9\times 10^{-4}, 1.1\times 10^{-4}, 0\rangle\ \text{N}}. \end{aligned}$$

7. (8 pts) You build a circuit with two alkaline cells, two short bulbs and four connecting wires, as shown in the diagram. You use a voltmeter to measure the voltages across each circuit element:

$V_B - V_A = 1.58$ volts; $V_C - V_B = -0.12$ volts. $V_E - V_D = -0.07$ volts;

$V_G - V_F = -0.16$ volts; $V_H - V_G = 1.56$ volts. $V_A - V_H = -0.05$ volts;

(a) (6 pts) The potential difference across the two identical bulbs are the same. What is the potential difference $V_D - V_C$? Give both magnitude and sign. *Show all steps in your work.*



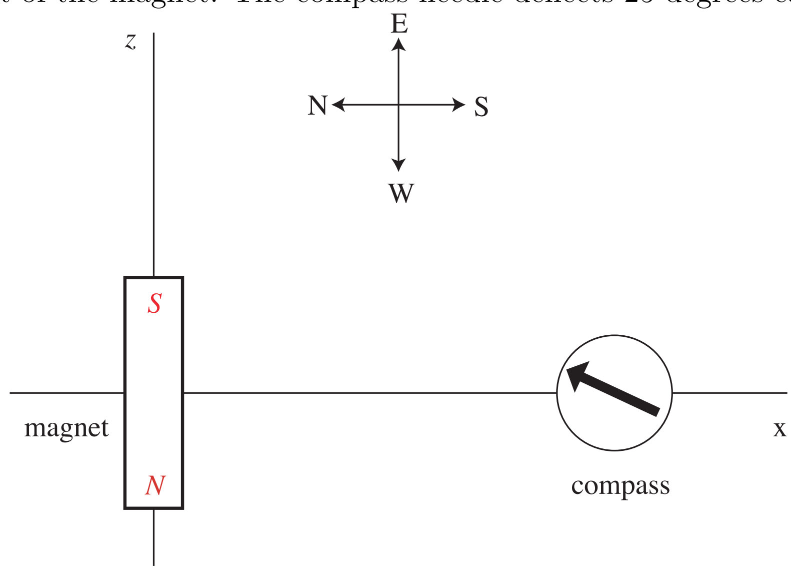
Since the potential difference across the two bulbs is the same, we have $V_F - V_E = V_D - V_C$. The round trip potential difference must be zero:

$$\begin{aligned}
\Delta V_{\text{round trip}} &= 0 \\
&= 1.58\text{ V} - 0.12\text{ V} + (V_D - V_C) - 0.07\text{ V} + (V_F - V_E) - 0.16\text{ V} \\
&\quad +1.56\ V - 0.05\ V \\
&= 2(V_D - V_C) + 2.74\text{ V} \\
\implies V_D - V_C &= \boxed{-1.37\text{ V}}.
\end{aligned}$$

(b) (2 pts) Your lab partner decides to verify the measurements. She connects one end of the black lead to the terminal labeled COM and one end of the red lead to the positive terminal on the voltmeter. She touches the other end of the black lead to the positive end of a single battery and the other end of the red lead to the negative end of the same battery (the battery between A and B). What reading will the voltmeter show now?

The voltmeter will read $\boxed{\text{-1.58 V}}$.

8. (12 pts) You are looking down on a magnet that lies on a table and is oriented along the z axis as shown. The diagram shows the top view. The horizontal component of the Earth's magnetic field is in the negative x direction. A compass is placed on the x axis to the right of the magnet. The compass needle deflects 25 degrees east of north.



(a) (2 pts) On the diagram, label the N and S poles of the bar magnet.

(b) (10 pts) The center of the magnet is 20 cm from the center of the compass. If the deflection is 25 degrees, determine the magnetic dipole moment of the magnet. *Show all steps in your work.*

The magnitude of the magnetic field due to the magnet at the compass is given by

$$B_{\text{magnet}} = B_{\text{Earth}} \tan(25^\circ) = 2 \times 10^{-5}\ \text{T} \times 0.47 = 9.33 \times 10^{-6}\ \text{T}.$$

We can relate this to the magnetic field due to a magnetic dipole on the perpendicular axis by

$$B_{dipole,\ \perp} = \frac{\mu_0}{4\pi}\frac{\mu}{r^3} \quad \Longrightarrow \quad \mu = \frac{r^3 B_{dipole,\ \perp}}{\mu_0/4\pi}.$$

Therefore,

$$\begin{aligned} \mu &= \frac{(0.20\ \text{m})^3 \times 9.33 \times 10^{-6}\ \text{T}}{10^{-7}\ \text{T} \cdot \text{m/A}} \\ &= \boxed{0.746\ \text{A}{\cdot}\text{m}^2}. \end{aligned}$$

Things you must know

Relationship between electric field and electric force

Conservation of charge

Electric field of a point charge

The Superposition Principle

Magnetic field of a moving point charge

Other Fundamental Concepts

$\Delta U_{el} = q\Delta V$

$\Delta V = -\int_i^f \vec{E}\cdot d\vec{l} \approx -\Sigma(E_x\Delta x + E_y\Delta y + E_z\Delta z)$

Specific Results

$\vec{E}$ due to uniformly charged spherical shell: *outside* like point charge; *inside* zero

$|\vec{E}_{dipole,\ axis}| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3}$ (on axis, $r \gg s$)

$|\vec{E}_{dipole,\ \perp}| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$|\vec{E}_{rod}| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2+(L/2)^2}}$ (r $\perp$ from center)

$|\vec{E}_{rod}| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r}$ (if $r \ll L$)

$|\vec{E}_{ring}| = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}$ (z alongaxis)

$|\vec{E}_{disk}| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$ (z alongaxis)

$|\vec{E}_{disk}| \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$|\vec{E}_{capacitor}| \approx \frac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$|\vec{E}_{fringe}| \approx \frac{Q/A}{\epsilon_0}\left(\frac{s}{2R}\right)$ (just outside capacitor)

$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\vec{r}}{r^2}$ (shortwire)

$|\vec{B}_{wire}| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2+(L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r}$ ($r \ll L$)

$|\vec{B}_{loop}| = \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{(z^2+R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$)

$\mu = IA = I\pi R^2$

$|\vec{B}_{dipole,\ axis}| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$ (on axis, $r \gg s$)

$|\vec{B}_{dipole,\ \perp}| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$i = nA\bar{v}$

$I = |q|nA\bar{v}$

$\Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$ (due to a point charge)

$E_{dielectric} = \frac{E_{applied}}{K}$

$K \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion : $\left|\frac{d\vec{p}_\perp}{d_t}\right| = \frac{|\vec{v}|}{R}|\vec{p}| \approx \frac{mv^2}{R}$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} C^2/(N $\cdot$ m^2)
Magnetic Constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T $\cdot$ m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T $\cdot$ m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T