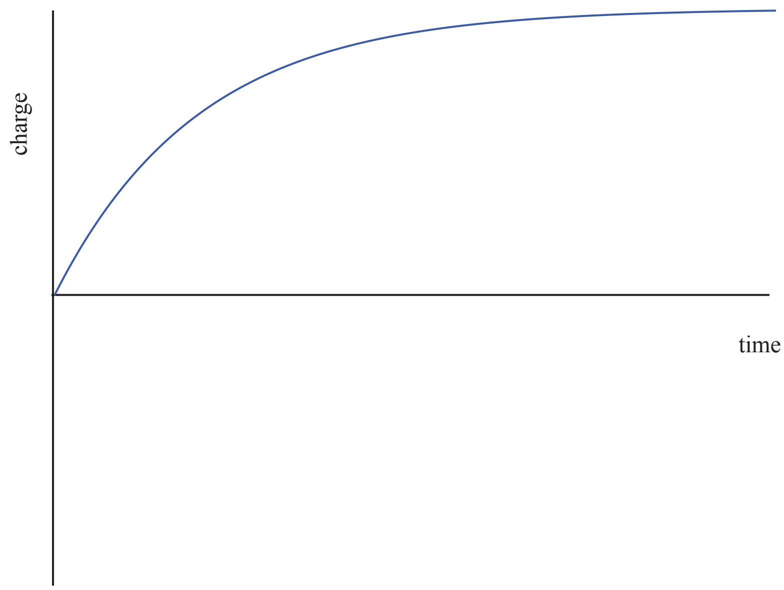
1. (8 pts) A particular capacitor has a separation between its plates of 0.03 mm. The area of one plate is 16 m^2. The capacitor is initially uncharged.

   (a) (4 pts) Draw a graph of the amount of charge on one plate of the capacitor vs. time, starting from the moment the capacitor is connected to a bulb and two ordinary 1.5 V batteries. Your graph should extend slightly past the time when the bulb goes out.



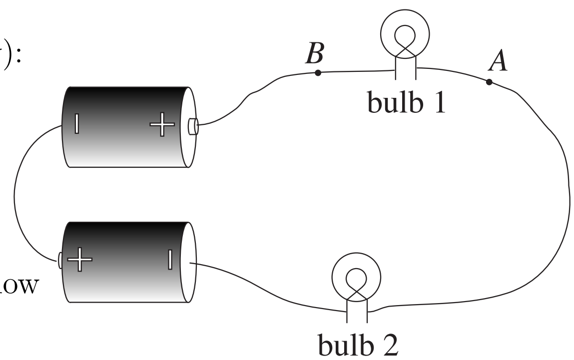
   (b) (4 pts) The circuit remains connected several minutes after the bulb has gone out. How much charge is now on the positive plate of the capacitor? Start from physics principles. Show your work clearly.

   The potential decrease across the capacitor will become the same as the potential increase across the batteries, which is 3.0 V. The electric field inside a capacitor

$$
\begin{aligned}
E &= \frac{Q/A}{\epsilon_0} = \frac{\Delta V}{d} \\
\Longrightarrow Q &= \frac{\epsilon_0 A \Delta V}{d} \\
&= \frac{8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)} \times 16\ \mathrm{m^2} \times 3.0\ \mathrm{V}}{0.03 \times 10^{-3}\ \mathrm{m}} \\
&= \boxed{1.4 \times 10^{-5}\ \mathrm{C}}.
\end{aligned}
$$

2. (20 pts) In the circuit shown here, bulbs 1 and 2 are identical in mechanical construction (both filaments have the same length of 4 mm and the same cross sectional area of 2.5e-8 m^2), but are made of different metals. The electron mobility in the metal used in bulb 2 is four times as large as the electron mobility in the metal used in bulb 1, but both metals have 6.3e28 mobile electrons/m^3. The two bulbs are connected in series to two batteries with thick copper wires. The emf of each battery is 1.5 volts.

a. (8 pts) Calculate the magnitude of the electric fields E_1 and E_2 inside each bulb. *Start from fundamental principles and show all steps in your work.*



The loop equation is ($\mathcal{E}$ = EMF of battery):

$$+2\mathcal{E} - E_1 L_1 - E_2 L_2 = 0,$$

where we are ignoring the small voltage drop in the thick copper wires. We also know that $L_1 = L_2 = L$.

The current equation is:

$$i_1 = i_2 \quad \Longrightarrow \quad n_1 A_1 u_1 E_1 = n_2 A_2 u_2 E_2.$$

We have $A_1 = A_2$, $n_1 = n_2$, $u_2 = 4u_1$, so

$$u_1 E_1 = u_2 E_2 = 4u_1 E_2 \quad \Longrightarrow \quad E_1 = 4E_2.$$

Putting everything together,

$$\begin{aligned} 2\mathcal{E} &= (E_1 + E_2)L = (4E_2 + E_2)L = 5E_2 L \\ \Longrightarrow \quad E_2 &= \frac{2\mathcal{E}}{5L} = \frac{2 \times 1.5\ \text{V}}{5 \cdot 4 \times 10^{-3}\text{m}} = 150\ \text{V/m}. \end{aligned}$$

So

$$\boxed{E_1 = 4\times\ 150\ \text{V/m} = 600\ \text{V/m}}, \qquad \boxed{E_2 = 150\ \text{V/m}}.$$

continued on next page

b. (4 pts) If the electron mobility of the metal in bulb 1 is 8e-5 (m/s)/(N/C), how many electrons per second pass through bulb 1? *Show all steps in your work.*

We know that

$$\begin{aligned} i &= nAu_1E_1 \\ &= 6.3 \times 10^{28}\ \frac{\text{electrons}}{\text{m}^3} \cdot 2.5 \times 10^{-8}\ \text{m}^2 \cdot 8 \times 10^{-5}\ \frac{\text{m/s}}{\text{V/m}} \cdot\ 600\ \text{V/m} \\ &= \boxed{7.6{\times}10^{19}\ \text{electrons/sec}}. \end{aligned}$$

c. (4 pts) What is the potential difference $V_B - V_A$, both magnitude and sign? *Show all steps in your work.*

$$\begin{aligned} V_B - V_A &= -\vec{E}_1 \cdot \Delta\vec{l}_1 \\ &= -(600\ \text{V/m})(-4 \times 10^{-3}\ \text{m}) \\ &= \boxed{+\ 2.4\ \text{V}}. \end{aligned}$$

d. (4 pts) What is the resistance of the wire in bulb 1? *Show all steps in your work.*

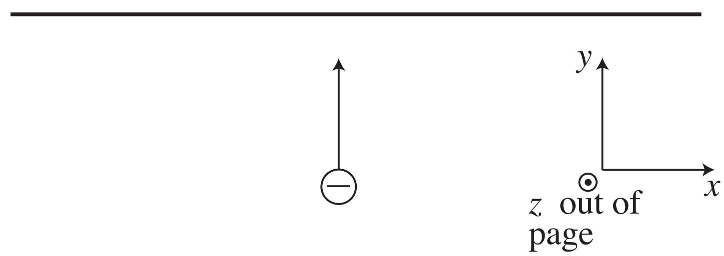
We know that

$$|\Delta V| = IR = |q|iR,$$

so

$$\begin{aligned} R &= \frac{|\Delta V|}{|q|i} \\ &= \frac{2.4\ \text{V}}{1.6 \times 10^{-19}\ \text{C} \cdot 7.6 \times 10^{19}\ \text{electrons/s}} \\ &= \boxed{0.20\ \Omega}. \end{aligned}$$

3. (14 pts) A current runs through a long straight wire which is oriented along the x-axis and passes through the origin. Conventional current runs in the +x direction. An electron is located at $\langle 0, -0.05, 0 \rangle$ m is moving with velocity $\langle 0, 8 \times 10^4, 0 \rangle$ m/s and experiences a magnetic force of magnitude 6.8×10^{-20} N due to the current in the wire.



a. (4 pts) What is the direction of the magnetic force on the electron (the moving charge outside the wire)? *Explain your reasoning in detail, explicitly using arrows to represent velocity, force and field vectors.*

The B field at the position of the charge is $\otimes$, which we get by grabbing on the wire with our right hand, with our thumb pointing in the +x direction.

We then have $\vec{v} \times \vec{B} = \uparrow \times \otimes = \leftarrow$.

The negative charge give the force then in the $\boxed{\text{+x direction}}$, $\rightarrow$.

b. (10 pts) What is the magnitude of the current in the wire? *Show all steps in your work.*

$$\vec{F} = q\vec{v} \times \vec{B} \quad \Longrightarrow \quad F = |q|vB\sin\theta = |q|vB,$$

since $\theta = 90°$. We also know that, since $r \ll L$,

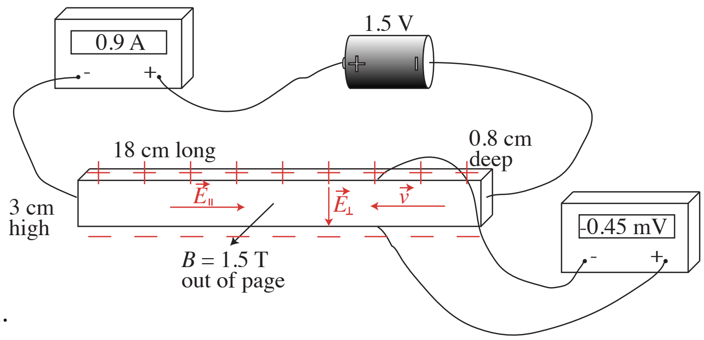
$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} \quad \Longrightarrow \quad F = \frac{\mu_0}{4\pi}\frac{2|q|vI}{r},$$

or

$$\begin{aligned} I &= \frac{F}{\mu_0/4\pi}\frac{r}{2|q|v} \\ &= \frac{6.8 \times 10^{-20}\ \text{N}}{10^{-7}\text{T} \cdot \text{m/A}}\frac{0.05\ \text{m}}{2 \cdot 1.6 \times 10^{-19}\ \text{C} \cdot 8 \times 10^4\ \text{m/s}} \\ &= \boxed{1.3\ \text{A}}. \end{aligned}$$

4. (20 pts) An experiment was carried out to determine the electrical properties of a new conducting material. A bar of the material, 18 cm long, 3 cm high and 0.8 cm deep, was inserted into a circuit with a 1.5 volt battery and carried a constant current of 0.9 ampere. The resistance of the copper connecting wires, ammeter, and the internal resistance of the battery, were all negligible compared to the resistance of the bar.

Using large coils not shown on the following diagram, a uniform magnetic field of 1.5 tesla was applied perpendicular to the bar (out of the page, as shown in the diagram below). A voltmeter was connected across the bar, with the connections across the bar carefully placed directly across from each other. The voltmeter reads -0.45 millivolts (-0.45×10^{-3} volt). Assume that there is only one kind of mobile charge in the bar material.



a. (2 pts) Draw an arrow indicating the direction of $\vec{E}_{\parallel}$, the electric field due to the battery and surface charges, which drives the the current in the bar. **Label this arrow $\vec{E}_{\parallel}$.**

b. (2 pts) Draw the distribution of charges on the surface of the bar due to the polarization of the bar by the magnetic force. Remember that a voltmeter gives a positive reading if the lead labeled "+" is connected to the higher potential location.

c. (2 pts) Draw an arrow indicating the direction of $\vec{E}_{\perp}$, the transverse electric field due to polarization of the bar by the magnetic force. **Label this arrow $\vec{E}_{\perp}$. Your answer must be consistent with part b.**

d. (3 pts) Are the mobile charges in the bar positive or negative? Explain carefully, using diagrams to support your explanation. *Your answer must be consistent with your answers in parts a, b, and c.*

If mobile charges are positive, $\vec{v} \times \vec{B} = (\rightarrow \times \odot) = \downarrow$, which contradicts polarization.

If mobile charges are negative, $-\vec{v} \times \vec{B} = -(\leftarrow \times \odot) = \downarrow$, which agrees with polarization.

So mobile charges in the bar are $\boxed{\text{negative}}$.

continued on next page

e. (2 pts) Draw an arrow indicating the direction $\vec{v}$, the drift velocity of the mobile charges in the bar. *Label this arrow* $\vec{v}$. *Your answer must be consistent with your answers in parts a, b, c, d.*

f. (3 pts) In the steady state, how long does it take for a mobile charge to go from one end of the bar to the other? *Show all steps in your work.*

The net force on the mobile charges is zero in the transverse direction, so

$$E_\perp = vB \quad \Longrightarrow \quad v = \frac{E_\perp}{B} = \frac{\Delta V/L}{B} = \frac{0.45 \times 10^{-3}\ \text{V}/0.03\ \text{m}}{1.5\ \text{T}} = 1.0 \times 10^{-2}\ \text{m/s}.$$

Therefore,

$$t = L/v = \frac{0.18\ \text{m}}{1.0 \times 10^{-2}\ \text{m/s}} = \boxed{18\ \text{s}}.$$

g. (3 pts) What is the mobility u of the mobile charges? *Show all steps in your work.*

The mobility is given by $u = \bar{v}/E_\parallel$, so we need $E_\parallel$. This is given by the loop equation:

$$\mathcal{E} - E_\parallel L = 0 \quad \Longrightarrow \quad E_\parallel = \frac{\mathcal{E}}{L} = \frac{1.5\ \text{V}}{0.18\ \text{m}} = 8.33\ \text{V/m}.$$

Therefore,

$$u = \frac{1.0 \times 10^{-2}\ \text{m/s}}{8.33\ \text{V/m}} = \boxed{1.20 \times 10^{-3}\ (\text{m/s})/(\text{V/m})}.$$

h. (3 pts) If each mobile charge is singly charged ($|q| = e$), how many mobile charges are there in 1 m^3 of this material? *Show all steps in your work.*

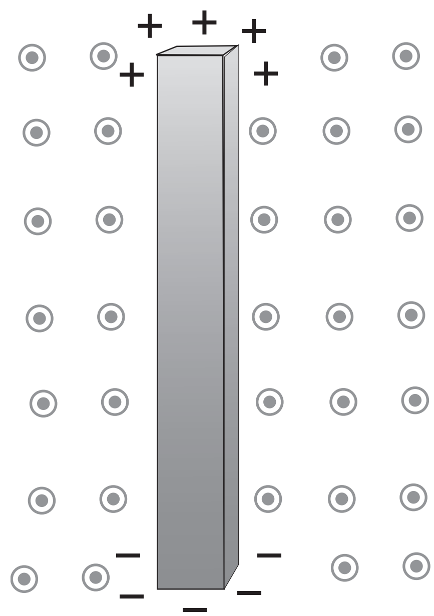
We can get this from the current. We have

$$I = |q|nA\bar{v}.$$

So

$$\begin{aligned} n &= \frac{I}{|q|A\bar{v}} \\ &= \frac{0.9\ \text{A}}{1.6 \times 10^{-19}\ \text{C} \cdot 0.008\ \text{m} \cdot 0.03\ \text{m} \cdot 1.0 \times 10^{-2}\ \text{m/s}} \\ &= \boxed{2.3 \times 10^{24}\ \text{mobile charges/m}^3}. \end{aligned}$$

5. (6 pts) A copper rod containing mobile electrons moves horizontally (in the $+x$ or $-x$ direction) at constant speed v through a region of uniform magnetic field directed out of the page. The rod is polarized as indicated in the diagram.



The following questions ask for ***brief*** explanations. Your explanations should contain enough information to explain your reasoning to another student who tried the problem but got a different answer from the one you got.

(a) (3 pts) What is the direction of the magnetic force on the mobile electrons in the copper bar? Explain briefly how you know this.

The magnetic force on the electrons is down (in the -y direction), since the electrons are collecting at the bottom, and lack of negative charge (i.e., positive charge) collects at the top. This occurs until the electric field is big enough to stop the flow of electrons.

(b) (3 pts) What is the direction of motion of the rod, $+x$ or $-x$? Explain how you know this. Include vectors in your explanation.

The force is in the -y direction, while the magnetic field is out of the page, or in the z direction, and $\vec{F} = q\vec{v} \times \vec{B}$. The charge is negative, so

$$-\hat{y} = -\hat{v} \times \hat{z},$$

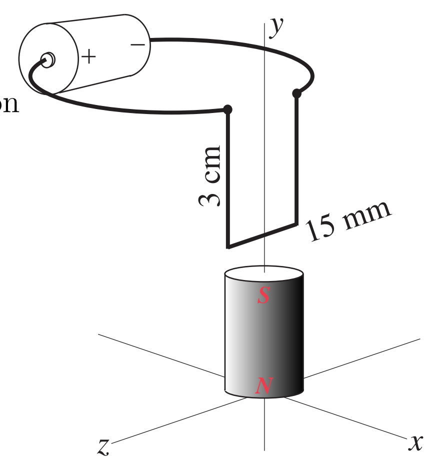
which means the velocity is in the $\boxed{-\hat{x} \text{ direction}}$.

6. (12 pts) A U-shaped piece of wire hangs from supports as shown, and is connected to a battery , as shown. The bottom piece of the wire is 15 mm long, and is oriented along the z axis. The vertical sides of the U are 3 cm long. Under the wire is a bar magnet, oriented along the y axis. When the circuit is connected, a conventional current of 7.5 A runs through the wire, and the 15 mm section of the wire is acted on by a force $\langle -4 \times 10^{-2}, 0, 0 \rangle$ N in the -x direction.

a. (2 pts) What is the direction of the conventional current through the 15 mm section of the wire?

Circle one: +x -x +y - y + z $\boxed{\text{-z}}$

b. (3 pts) On the diagram label the North and South poles of the bar magnet. *Explain clearly how you decided, using diagrams that include vectors.*



We have (for the directions, ignoring the magnitudes) $\vec{F} = -\hat{x}$, $\Delta\vec{l} = -\hat{z}$, and the equation is $\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$, so we need

$$-\hat{x} = -\hat{z} \times \hat{B}.$$

We therefore get $\vec{B}$ in the $-\hat{y}$ direction, which means the north pole of the magnet is down.

c. (7 pts) What is the magnitude of the magnetic field at the location of the 15 mm length of wire, due to the bar magnet? *Show all steps in your work.*

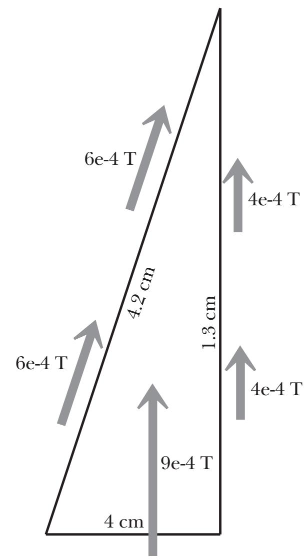
We have

$$|\vec{F}| = I|\Delta\vec{l} \times \vec{B}| = I\Delta l B \quad \Longrightarrow \quad B = \frac{F}{I\Delta l}.$$

So

$$B = \frac{4 \times 10^{-2}\ \text{N}}{7.5\ \text{A} \times 0.015\ \text{m}} = \boxed{0.36\ \text{T}}.$$

7. (14 pts) You measure the magnetic field all around a triangular path and find the pattern shown here.



(a) (3 pts) What is the integral of the magnetic field along the right side? Show work.

I will integrate counterclockwise. So the integral along the right side is

$$\begin{aligned}\int \vec{B}\cdot d\vec{l} &= |B|L\cos\theta \\ &= 4\times 10^{-4}\ \mathrm{T}\times 1.3\times 10^{-2}\ \mathrm{m}\cos(0) \\ &= \boxed{5.2\times 10^{-6}\ \mathrm{T}\cdot\mathrm{m}}.\end{aligned}$$

(b) (3 pts) What is the integral of the magnetic field along the left side? Show work.

Now we have

$$\begin{aligned}\int \vec{B}\cdot d\vec{l} &= |B|L\cos\theta \\ &= 6\times 10^{-4}\ \mathrm{T}\times 4.2\times 10^{-2}\ \mathrm{m}\cos(180^\circ) \\ &= \boxed{-2.52\times 10^{-5}\ \mathrm{T}\cdot\mathrm{m}}.\end{aligned}$$

(c) (3 pts) What is the integral of the magnetic field along the bottom side? Show work.

This time we have

$$\begin{aligned}\int \vec{B}\cdot d\vec{l} &= |B|L\cos\theta \\ &= 9\times 10^{-4}\ \mathrm{T}\times 4\times 10^{-2}\ \mathrm{m}\cos(90^\circ) = \boxed{0\ \mathrm{T}\cdot\mathrm{m}}.\end{aligned}$$

(d) (5 pts) What is the magnitude and direction of the conventional current passing through the triangle? Show your work.

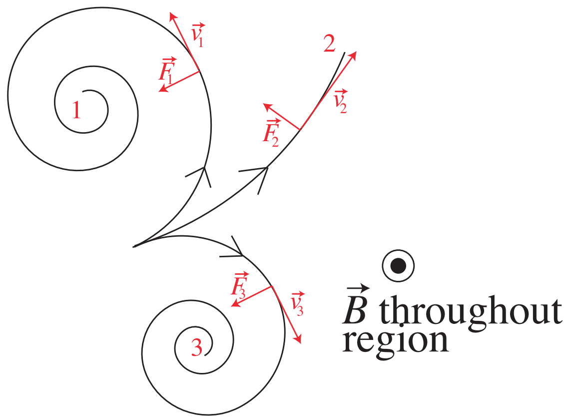
We have

$$\begin{aligned}\oint \vec{B}\cdot d\vec{l} &= \mu_0 I \\ \Longrightarrow I &= \frac{1}{\mu_0}\oint \vec{B}\cdot d\vec{l} \\ &= \frac{1}{4\pi\times 10^{-7}\ \mathrm{T}\cdot\mathrm{m/A}}\left(5.2\times 10^{-6}\ \mathrm{T}\cdot\mathrm{m} - 2.52\times 10^{-5}\ \mathrm{T}\cdot\mathrm{m}\right) = -16\mathrm{A},\end{aligned}$$

or $\boxed{\text{16 A into the page}}$.

8. (6 pts) Charged particles traveling through a bubble chamber leaves visible trails behind them. A photograph taken in a bubble chamber showed the tracks of three particles. The particles were traveling in the directions indicated by arrows on the diagram, and there was a strong uniform magnetic field pointing out of the page throughout the region.

   On the diagram, identify the sign of the charge of each particle. *Explain your reasoning below, explicitly using arrows to represent velocity, field, and force vectors.*



   Using the right-hand rule,

   - For particle 1: $\vec{v}_1 \times \vec{B}$ will give $\vec{F}_1$ as shown for a negative charge.
   - For particle 2: $\vec{v}_2 \times \vec{B}$ will give $\vec{F}_2$ as shown for a negative charge.
   - For particle 3: $\vec{v}_3 \times \vec{B}$ will give $\vec{F}_3$ as shown for a positive charge.

Things you must know

Relationship between electric field and electric force

Electric field of a point charge

Magnetic field of a moving point charge

Conservation of charge

The Superposition Principle

Other Fundamental Concepts

$$\Delta U_{el} = q\Delta V$$

$$\Delta V = -\int_i^f \vec{E}\cdot d\vec{l} \approx -\Sigma(E_x\Delta x + E_y\Delta y + E_z\Delta z)$$

$$\Phi_{\text{el}} = \int \vec{E}\cdot\hat{n}\,dA$$

$$\Phi_{\text{mag}} = \int \vec{B}\cdot\hat{n}\,dA$$

$$\oint \vec{E}\cdot\hat{n}\,dA = \frac{\sum q_{\text{inside}}}{\epsilon_0}$$

$$\oint \vec{B}\cdot\hat{n}\,dA = 0$$

Ampere without Maxwell (no displacement current) $\oint \vec{B}\cdot d\vec{l} = \mu_0 \sum I_{\text{inside path}}$

Specific Results

$\vec{E}$ due to uniformly charged spherical shell: *outside* like point charge; *inside* zero

$$|\vec{E}_{dipole,\ axis}| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3} \text{ (on axis, } r \gg s)$$

$$|\vec{E}_{dipole,\ \perp}| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3} \text{ (on } \perp \text{ axis, } r \gg s)$$

$$|\vec{E}_{rod}| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2+(L/2)^2}} \text{ (} r \perp \text{ from center)}$$

$$|\vec{E}_{rod}| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r} \text{ (if } r \ll L)$$

Electric dipole moment $p = qs$

$$|\vec{E}_{ring}| = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2+R^2)^{3/2}} \text{ (} z \text{ alongaxis)}$$

$$|\vec{E}_{disk}| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right] \text{ (} z \text{ alongaxis)}$$

$$|\vec{E}_{disk}| \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q/A}{2\epsilon_0} \text{ (if } z \ll R)$$

$$|\vec{E}_{capacitor}| \approx \frac{Q/A}{\epsilon} \text{ (} +Q \text{ and } -Q \text{ disks)}$$

$$|\vec{E}_{fringe}| \approx \frac{Q/A}{\epsilon}\left(\frac{s}{2R}\right) \text{ (just outside capacitor)}$$

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\vec{r}}{r^2} \text{ (shortwire)}$$

$$|\vec{B}_{wire}| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2+(L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r} \text{ (} r \ll L)$$

$$|\vec{B}_{loop}| = \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{(z^2+R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{z^3} \text{ (on axis, } z \gg R)$$

$$\mu = IA = I\pi R^2$$

$$|\vec{B}_{dipole,\ axis}| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3} \text{ (on axis, } r \gg s)$$

$$|\vec{B}_{dipole,\ \perp}| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3} \text{ (on } \perp \text{ axis, } r \gg s)$$

$$i = nA\bar{v} \qquad I = |q|nA\bar{v} \qquad \bar{v} = uE$$

$$\sigma = |q|nu \qquad J = \frac{I}{A} = \sigma E \qquad R = \frac{L}{\sigma A}$$

$$E_{dielectric} = \frac{E_{applied}}{K} \qquad \Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right] \text{ (due to a point charge)}$$

$$Q = C|\Delta V| \qquad \text{Power} = I\Delta V \qquad I = \frac{|\Delta V|}{R} \text{ (ohmic resistor)}$$

$$K \approx \frac{1}{2}mv^2 \text{ if } v \ll c \qquad \text{circular motion : } \left|\frac{d\vec{p}_\perp}{d_t}\right| = \frac{|\vec{v}|}{R}|\vec{p}| \approx \frac{mv^2}{R}$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N · m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} C^2/(N · m^2)
Magnetic Constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T · m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T · m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T