Dec 5: Get Whiteboards and Clickers

Clicker questions

Q1: We have two blocks, one aluminum (Al) and one lead (Pb), each containing $6 \times 10^{23}$ atoms (one mole). The aluminum block has a mass of 27 grams, and the lead block has a mass of 207 grams. Which of the following pictures shows the blocks in the correct relative sizes?

A) [Image of Al and Pb blocks]
B) [Image of Al and Pb blocks]
C) [Image of Al and Pb blocks]
Initially the two blocks are at a temperature very near absolute zero (0 K). We will add 1 J of energy to the aluminum block, and 1 J of energy to the lead block, and see which block has the larger increase in temperature. We will step through a chain of reasoning using statistical mechanics to answer this question, which will let us determine whether aluminum or lead has the higher heat capacity at low temperatures.
Q2: From Young’s modulus we found that the effective stiffness of the interatomic bond for Al is about 16 N/m and for Pb is about 5 N/m. A mole of Al is 27 grams, and a mole of Pb is 207 grams. Here are energy level diagrams for the quantized harmonic oscillators used in the Einstein solid. Which diagram represents Al and which represents Pb?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) (A) is Al and (B) is Pb</td>
<td>B) (A) is Pb and (B) is Al</td>
</tr>
<tr>
<td>C) (B) is both Al and Pb (they are the same)</td>
<td></td>
</tr>
</tbody>
</table>

\[ \Delta E = h \omega \]
\[ \omega = \sqrt{\frac{k}{m}} \]

\[
\begin{align*}
\text{4.16} & \text{ vs } \frac{4.5}{287} \\
\frac{4.5}{287} & \text{ vs }
\end{align*}
\]
Q3: We add 1 J of energy to each block. Given the fact that Al has the greater energy-level spacing, which block now has the larger number of quanta of energy, \( q \)?

A) The number of quanta \( q \) is greater in the Al

B) The number of quanta \( q \) is greater in the Pb

C) The number of quanta \( q \) is the same for Al and Pb
Q4: What about the number $N$ of quantized oscillators in the two blocks?
A) $N$ is greater in the Al
B) $N$ is greater in the Pb
C) $N$ is the same for Pb and Al
Q5: The Pb block has more quanta corresponding to the 1J of thermal energy. Therefore, in which block is there a larger number of ways \( \Omega \) of arranging the thermal energy?

A) The number of ways \( \Omega \) is greater in the Al

B) The number of ways \( \Omega \) is greater in the Pb

C) The number of ways \( \Omega \) is the same in the Pb and the Al

\[
\Omega = \frac{(g+n-1)!}{g!(n-1)!}
\]
Q6: The Pb block has the larger number of ways \( \Omega \) to arrange the energy. So which block now has the larger entropy \( S \)?

A) The entropy \( S \) is now greater in the Al

B) The entropy \( S \) is now greater in the Pb

C) The entropy \( S \) is the same in the Al and the Pb

\[ S = k \ln \mathcal{R} \]
Q7: Originally the temperature of the blocks was near absolute zero, with almost no thermal energy in the blocks. How many ways are there to arrange zero energy in a block? Just 1. So what was the original entropy in a block?
A) 0 J/K
B) 1 J/K
C) infinite

\[ S = k \ln \Omega \]

\[ = k \ln 1 = 0 \]
Q8: We found that after adding 1 J to each block, the entropy $S$ is now greater in the Pb block. Both blocks started with zero entropy. Therefore which block experienced a larger change in entropy $\Delta S$?

A) The entropy change $\Delta S$ was greater in the Al

B) The entropy change $\Delta S$ was greater in the Pb

C) The entropy change $\Delta S$ was the same in the Pb and the Al
Q9: We added the same amount of energy $\Delta E = 1 \text{ J}$ to each block, and the entropy change $\Delta S$ was greater in the Pb block. Which block now has the higher temperature?

A) The temperature of the Al is now higher
B) The temperature of the Pb is now higher
C) The temperature of the Al and Pb are the same

$$\frac{1}{T} = \frac{\Delta S}{\Delta E}$$
Q10: The original temperature was 0 K, and the final temperature of the Al block is higher than that of the Pb block, so the Al block has the larger *change* in temperature, $\Delta T$. At low temperatures, which block has the greater heat capacity per atom, $C = (\Delta E/\Delta T)/6e23$?
A) The low-temperature heat capacity per atom of Al is greater
B) The low-temperature heat capacity per atom of Pb is greater
C) The low-temperature heat capacity per atom is the same for Pb and Al

$$C = \frac{\Delta E}{\Delta T}$$
Here are actual heat capacity data for Al and Pb (see textbook):
Boltzmann Distribution $e^{-\frac{E}{kT}}$

Problem 11.X.12, pg 398.

Problem 11.X.13 Marbles on desk

Problem 11.X.14

Ideal gas

$(e^{-\frac{K_{\text{trans}}}{kT}}) (e^{-\frac{E_2}{kT}}) (e^{-\frac{E_3}{kT}}) (e^{-\frac{H_{\text{coll}}}{kT}})$
An air molecule gets bumped off desk at room temp. How high does it go if it does not hit any other molecules?

In "typical" height when $\frac{Mg}{kT} = 1$

$$y = \frac{1}{2g} = \frac{(1.38 \times 10^{-23} \text{ J/k}) / (300 \text{ K})}{\left( \frac{6.029 \times 10^{23} \text{ mol}^{-1}}{9.8 \text{ m/s}^2} \right) / \left( 6 \times 10^{-23} \text{ m}^2 \text{mol}^{-1} \right)} = 8.7 \text{ km}$$
Marble on a desk, \( M = 10 \text{g} \). What is the typical height of a marble, i.e., \( M g y = 6T \)?

\[
y = \frac{(1.38 \times 10^{-23} \text{ J/K}) \times 300 \text{ K}}{(0.01 \text{ kg}) \times 9.8 \text{ m/s}^2} = 4 \times 10^{-8} \text{ m}
\]
Problem 11.X.14

Approximate what fraction of sea-level air density is found atop Mt. Everest?

$h = 8848 \text{ m above sea level}$

\[ P = P_{\text{sea-level}} e^{-\frac{M_g y}{kT}} \]

\[ \frac{P}{P_{\text{sea-level}}} = e^{-\frac{0.028 \times 8848}{6.105 \times 10^{-7} \times 273}} = e^{-3.35} \approx 0.35 \]
5000 J added to system, increasing its entropy by 50 J/k

Found temp to be \( T = \frac{\Delta E}{\Delta S} = \frac{5000 J}{50 J/k} = 100 K \)

Different system, we add 5000 J, its entropy increased by 500 J/k. What’s temp?

\[ T = \frac{\Delta E}{\Delta S} = \frac{5000 J}{500 J/k} = 10 K \]

Put in thermal contact, let 5000 J flow from hot to cold. What is change in entropy of total system?
What is change in entropy?

\[
\frac{1}{T} = \frac{\Delta S}{\Delta E}
\]

\[
\Delta S_{100K} = \frac{\Delta E}{T} = \frac{-5000J}{100K} = -50J/K
\]

\[
\Delta S_{10} = \frac{\Delta E}{T} = \frac{+5000J}{10K} = 500J/K
\]

\[
\Delta S = \Delta S_{100} + \Delta S_{10} = 450J/K > 0
\]
Q1. A certain object (for which the Einstein solid is not a good model) has entropy \( S = bE^{1/2} \), where \( b \) is a constant. We know that \( \frac{1}{T} = \frac{dS}{dE} \). Find the temperature \( T \) as a function of energy \( E \).

<table>
<thead>
<tr>
<th>Entropy ( S = bE^{1/2} ):</th>
<th>( \frac{1}{T} = \frac{dS}{dE} = \frac{d(bE^{1/2})}{dE} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) ( T = \frac{2}{b} E^{1/2} )</td>
<td>( = \frac{b}{2} E^{-1/2} )</td>
</tr>
<tr>
<td>B) ( T = \frac{b}{2} E^{-1/2} )</td>
<td></td>
</tr>
<tr>
<td>C) ( T = \frac{b}{2} E^{1/2} )</td>
<td></td>
</tr>
<tr>
<td>D) ( T = bE^{-1/2} )</td>
<td></td>
</tr>
<tr>
<td>E) ( T = bE^{1/2} )</td>
<td></td>
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</tbody>
</table>

\[ T = \frac{2}{b} E^{1/2} \]
Q2. Suppose the object has a mass of 1 gram. Calculate the heat capacity on a per-gram basis as a function of the temperature \( T \), starting from temperature \( T = \frac{2}{b} E^{1/2} \):

<table>
<thead>
<tr>
<th>A) ( b^2 T )</th>
<th>D) ( \frac{b^2}{4} T^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B) ( \frac{b^2}{2} T )</td>
<td>E) ( \frac{b}{2} T )</td>
</tr>
</tbody>
</table>

\[
C = \frac{dE}{dT}
\]

\[
T^2 = \frac{4}{b^2} E
\]

\[
E = \frac{b^2 T^2}{4}
\]

\[
\frac{dE}{dT} = \frac{d}{dT} \left( \frac{b^2 T^2}{4} \right)
\]

\[
C = \frac{b^2 T}{2}
\]