Tuesday 10/16/07

Ponderable

Initially $E_i = m_n c^2$
Finally $E_f = m_p c^2 + K_p + m_e c^2 + K_e + K_{\text{antineutrino}}$ (\(m_{\text{antineutrino}}\) very small but not quite zero)
Since the $K$’s aren’t zero, that says $m_n > m_p + m_e$ so less mass than when you started.

\[
m_p c^2 + K_p + m_e c^2 + K_e + K_{\text{antineutrino}} = m_n c^2 + W
\]
(938.3 MeV) + $K_p$ + (0.511 MeV) + $K_e$ + $K_{\text{antineutrino}}$ = (939.6 MeV) + 0
(938.8 MeV) + $K_p$ + $K_e$ + $K_{\text{antineutrino}}$ = (939.6 MeV) + 0
$K_p + K_e + K_{\text{antineutrino}} = 0.8$ MeV = 1.28e-13 J

That’s what happens in fission and fusion reactions. Reaction products less massive than starting particles. Lots of energy output.

At first scientists didn’t see antineutrino. They thought energy principle didn’t work. Pauli predicted, but not seen for 30 years.

Ponderable: It’s rocket science!

Consider a rocket blasting off from a planet. (Assume the launch happens very quickly and then the rocket coasts away from the planet.) If we know the launch speed, what is the rocket’s final speed for some distance $r$ from the planet’s center? We could use the momentum principle, but the force varies—we’d have to integrate or do it on a computer. Since distance is involved, we should more likely use the energy principle. We will pick the system to be both the planet and rocket so no external work is done (and thus we don’t have to worry about integrating the force.)
\[ E_f = E_i + W = E_i + 0 \]
\[ \left( Mc^2 + K_M + mc^2 + K_m + U_{Mm} \right)_f = \left( Mc^2 + K_M + mc^2 + K_m + U_{Mm} \right)_i \]
\[ (K_m + U_{Mm})_f = (K_m + U_{Mm})_i \]
\[ \frac{1}{2} m |\vec{v}|^2 - G \frac{Mm}{r_f} = \frac{1}{2} m |\vec{v}_i|^2 - G \frac{Mm}{r_i} \quad \text{since } \vec{v} \ll c \]
\[ |\vec{v}_f| = \sqrt{ |\vec{v}_i|^2 + 2GM \left( \frac{1}{r_f} - \frac{1}{r_i} \right) } \]

We are assuming the planet’s kinetic energy \( K_M \) doesn’t change much. The mathematical calculation above was straightforward, so let’s also consider how things look graphically.

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**Discussion**

Plot \( U \) vs. \(|\vec{r}|\)

\[
U = -\frac{G M m}{|\vec{r}|} \quad \text{As } |\vec{r}| \to \infty, \quad U = -\frac{G M m}{|\vec{r}|}, \text{ which is negative, gets closer to zero}
\]

\( |\vec{r}| \) is planet’s radius

\[ F_r = -\frac{dU}{dr} = G \frac{Mm}{r^2}, \text{ which is negative of the slope} \]

Plot \( K \) vs. \(|\vec{r}|\)

\( K \) is decreasing due to the planet pulling on the rocket. The way we’ve drawn this graph, the rocket never actually stops moving. As the gravitational force gets very small, the rate of change of the rocket’s momentum becomes negligible—the speed becomes nearly constant and so does \( K \).

Because there are no external forces acting on the system, the total energy \( K + U \) is constant. That tells us the shape of the \( K \) curve is just like that of \( U \), except “flipped over” so that the two curves added up will produce a horizontal line corresponding to the constant energy of the system.
Plot a separate graph with both $K$ and $U$ on it. It is easiest to plot one point at large $|\vec{r}|$, where $U$ is near zero, so $K + U$ is just under the $K$ graph.

What would it look like with a smaller initial speed, so that $K + U$ is negative? The rocket will eventually slow to a stop and then fall back to the planet. This is called a bound state. The area to the right of the red and blue lines is called the forbidden region. Again, the easy way to do this is to pick some $r$ such as the turnaround point. Here $K = 0$, so $K + U$ is just the value of $U$ at this point.
What happens if the launch speed is “just right” so that $K$ approaches 0 at a very, very large distance from the planet? Since we know $U$ goes to zero when we are infinitely far from the planet, so must $K + U$. Since this is a constant value, the total energy is zero for any distance $|\vec{r}|$ from the planet.

$$K_i + U_i = 0$$
$$\frac{1}{2}m|\vec{v}_i|^2 - G\frac{Mm}{R} = 0$$
$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

In this borderline case, where the initial speed is just big enough to make $K + U$ not negative (so this is not a bound state) the initial speed is called the “escape speed.” For Earth, $v_{\text{escape}} = 11$ km/s.
Consider two protons that are initially a distance $|\vec{r}|$ apart.

The electrical force is very similar in form to the gravitational force:

$$|\vec{F}_{elec}| = \frac{1}{4\pi\varepsilon_0} \frac{|Qq|}{|\vec{r}|^2} = \left( 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \right) \left( \frac{1.6 \times 10^{-19} \text{C}^2}{|\vec{r}|^2} \right)$$

This gives us the electrical potential energy as:

$$U = + \frac{1}{4\pi\varepsilon_0} \frac{|Qq|}{|\vec{r}|}$$

Since there are again no external forces, the total system energy is constant. Start by graphing the $U$ function, which is positive this time. It still approaches zero as the separation increases. When the separation is very small, the potential energy is very large. As the protons move away from each other, their kinetic energies rise, and their interaction energy (electric potential energy) falls. Algebraically

$$E_f = E_i + W$$

$$m_1c^2 + K_{1f} + m_2c^2 + K_{2f} + U_f = m_1c^2 + K_{1i} + m_2c^2 + K_{2i} + U_i + 0$$

$$K_{1f} + K_{2f} + U_f = U_i$$

$$K_{1f} + K_{2f} = K_f = U_i$$

$$U_f = 0$$

$K + U = \text{constant}$

$U = + \frac{1}{4\pi\varepsilon_0} \frac{|Qq|}{|\vec{r}|}$