RELAXING NEAR THE CRITICAL POINT

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The big problem

What are the properties and signatures of the quark-hadrons phase transition ?

too difficult

take an effective scalar model with $O(N)$ global symmetry (pions)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{1}{2} m^2 \vec{\phi}^{\,2} - \frac{\lambda}{2N} (\vec{\phi}^{\,2})^2 \,,$$

$$\vec{\phi} = (\phi^1, \ldots, \phi^N) \,, \quad m^2 < 0 \,.$$

Phenomenologically $N = 4$ but take $N \to \infty$

Then we can ask non-perturbative questions.

The specific question

Consider a near equilibrium system

$$n_k(t) = \frac{1}{\exp(\omega_k/T) - 1} + \delta n_k(t)$$

near the critical temperature T_c defined by

$$\lim_{T \to T_c} \xi(T) = \infty \, .$$

The simplest expectation is

$$\delta n_k(t) = e^{-2\Gamma_k(T)t} \delta n_k(0) \, ;$$

then it is possible to define a relaxation time

$$\tau_k(T) = \frac{1}{\Gamma_k(T)} \, .$$

Consider low-energy particles ($k \to 0$) and a temperature $T \to T_c$; is then

$$\lim_{k \to 0} \lim_{T \to T_c} \Gamma_k(T) = 0 \, ?$$

If yes, there is critical slowing down i.e. long wavelenght fluctuations require an extremely long time to thermalize. This is the case in condensed matter physics.

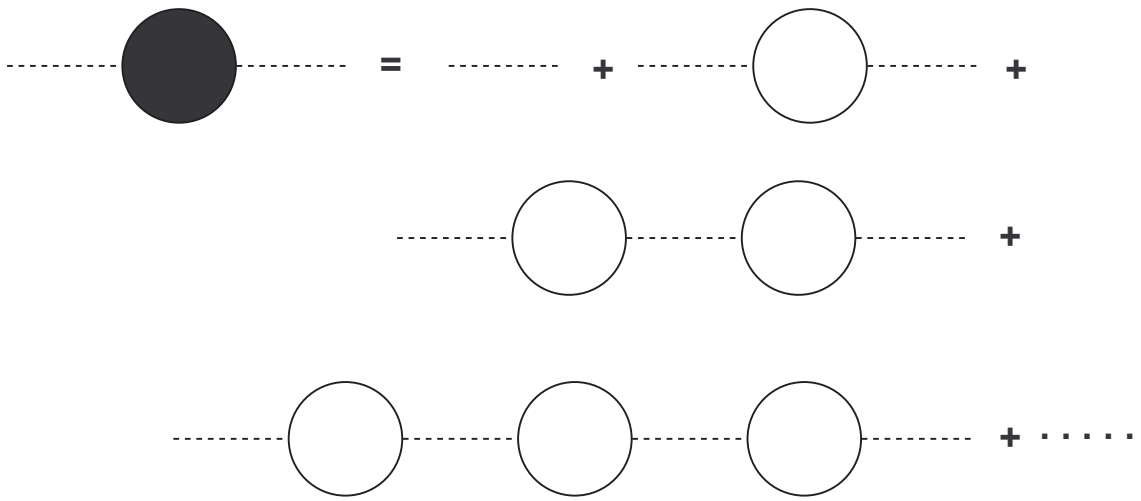
Quantum Field Theory Analysis

Non-trivial since at $k \to 0$ and $T \to T_c$ one expects IR singularities invalidating perturbation theory. However one can invoke the large N expansion to resum contributions to all orders in λ.

Lagrangian:

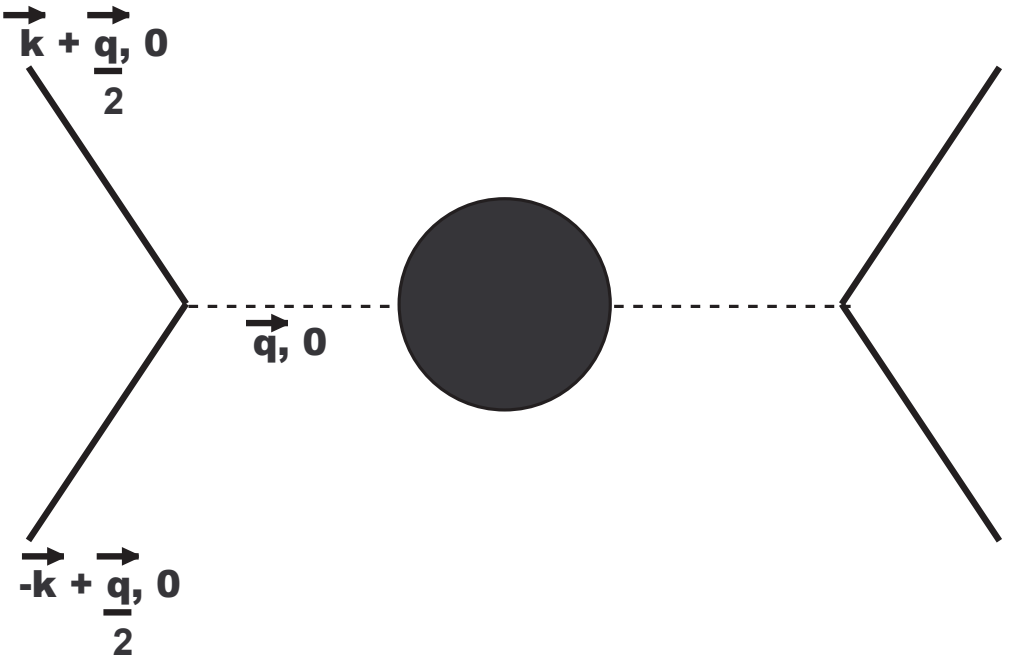
$$\mathcal{L} = \frac{1}{2}\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - \frac{1}{2} m^2 \vec{\phi}^{\,2} - \sqrt{\frac{\lambda}{N}} \chi \vec{\phi}^{\,2} + \frac{1}{2}\chi^2$$

χ field resummed propagator:



Review of the static case

$$\lambda_{eff}(q) = \frac{\lambda}{1 + I_{bub}(0, q)}$$



$$q \ll T = T_c, \qquad I_{bub}(0, q) = \frac{\lambda T}{4q} \,,$$

$$\Rightarrow \qquad \lambda_{eff}(q) \simeq \frac{4q}{T} \to 0, \qquad q \to 0$$

infrared screening of the interaction

What we are not doing

- Three-dimensional effective theory

  For static phenomena and soft momenta $q \ll T = T_c$ one can define the 3D coupling $\lambda_3(q) = \frac{\lambda T}{4q}$ and the 3D effective coupling

  $$\lambda_{3,eff}(q) = \frac{\lambda_3(q)}{1 + \lambda_3(q)}$$

  which has a non-trivial IR fixed point. However the three-dimensional language is not useful for the analysis of dynamical quantities and will not be used here.

- Classical approximation

  If the damping rate integral were dominated by soft momenta $q \ll T$ one could replace $n_q \to \frac{T}{q}$. However the classical approximation fails in the regime relevant for the critical slowing down and will not be used here.

The damping rate at next-to-leading order

We compute the damping rate from linear response theory

$$\ddot{\varphi}_k(t)+\omega_k^2\ \varphi_k(t)+\int_{-\infty}^{\infty}\Sigma_{ret,k}(t-t')\ \varphi_k(t')\ dt' = J_k(t)$$

where

$$\Sigma_{ret,k}(t-t') = \int_{-\infty}^{\infty}\tilde{\rho}(\omega,k)\sin\Big[\omega(t-t')\Big]\ d\omega$$

and

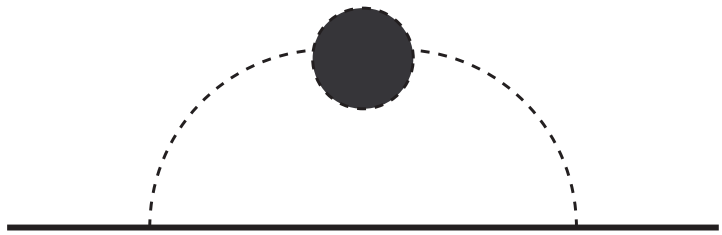
$$\omega_k^2 = k^2 + m_T^2, \quad m_T^2 = \delta M^2(T) + M_{tad}^2(T)$$

The thermal mass m_T is defined from the gap equation solved at leading order in $1/N$ whereas the damping rate is defined as

$$\Gamma_k(T) = -\frac{\pi\tilde{\rho}(\omega_k,k)}{2\omega_k}$$

and computed at next-to-leading order in $1/N$.

Explicit expression at $T = T_c$

$\tilde{\rho}(\omega_k, k) \sim \text{Im}$  $=$

$$-\frac{4\lambda}{N}\int \frac{d^3q}{(2\pi)^3}\frac{dq_0}{2|\vec{k}+\vec{q}|}\,\rho(q_0,q)\left[1+n_{\vec{q}+\vec{p}}+n(q_0)\right]\times$$

$$\left[\delta(\omega-q_0-|\vec{k}+\vec{q}|)-\delta(\omega+q_0+|\vec{k}+\vec{q}|)\right]\ ,$$

$$\rho(q_0,q)=\frac{1}{\pi}\,\frac{\Pi_I(q_0,q)}{[1+\Pi_R(q_0,q)]^2+\Pi_I^2(q_0,q)}$$

$$\Pi_I(q_0,q)=\frac{\lambda T}{4\pi q}\ln\left[\frac{1-e^{-\frac{|q_0+q|}{2T}}}{1-e^{-\frac{|q_0-q|}{2T}}}\right]+\mathcal{O}\left(\lambda\, T^0\right)$$

$$\Pi_R(q_0,q)=\frac{\lambda T}{4q}\left[\Theta(q-q_0)-\Theta(-q-q_0)\right]+\mathcal{O}\left(\lambda\,\ln T\right)$$

The dominant integration region

For soft momenta $k < \lambda T$, the damping rate integral is dominated by semihard momenta $q > \lambda T/4$; in this regime the bubble spectral density can be approximated as

$$\rho(k \pm |\vec{k}+\vec{q}|, q) \simeq \frac{\frac{4q}{\lambda T} L_{\pm}}{\pi^2 \left[\frac{4q}{\lambda T}\right]^2 + L_{\pm}^2}$$

with

$$L_{\pm} = \log \left| \frac{k \pm |\vec{k}+\vec{q}| + q}{k \pm |\vec{k}+\vec{q}| - q} \right| .$$

We see the emergence of a non-perturbative scale

$$q^*(k,T) = \frac{\lambda T}{4\pi} \ln \frac{\lambda T}{4\pi k} \quad (q^* > T \text{ or } q^* < T)$$

such as the logarithm is of the same order of the power term $\left[\frac{4q}{\lambda T}\right]^2$.

The dominant integration region is $q \approx q^*$, where $\rho \sim 1$.

Small coupling regimes ($\frac{\lambda}{4\pi} \ll 1$)

If we define the ultrasoft scale

$$k_{us}(\lambda, T) = \frac{\lambda T}{4\pi} \exp(-4\pi/\lambda)$$

then:

q^* is soft ($q^* < T$) for $k > k_{us}$ (cl. approx. OK)

q^* is hard ($q^* > T$) if $k < k_{us}$ (NO cl. approx.)

There are three regimes:

1. Semisoft k: $\lambda T < k < T$

2. Soft k: $\frac{\lambda T}{4\pi} \exp(-4\pi/\lambda) < k < \lambda T$

3. Ultrasoft k: $k < \frac{\lambda T}{4\pi} \exp(-4\pi/\lambda)$

In regime 1 $(\lambda T < k < T)$ perturbation theory is reliable and the two-loops result is

$$\Gamma_k(T) = \lambda^2 \frac{N+2}{8\pi^2 N^2} \frac{T^2}{k} \to \infty$$

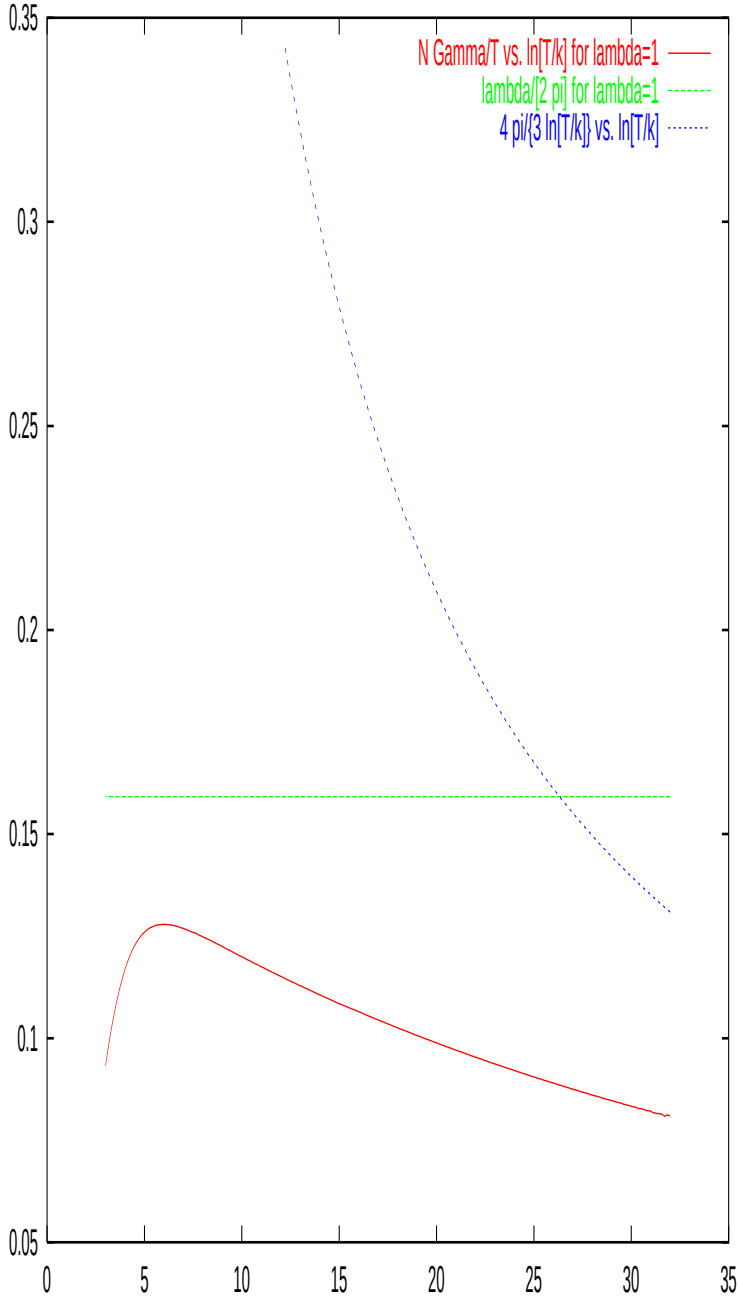
In regime 2 $(k_{us} < k < \lambda T)$ large N resummation is needed but the classical approximation is reliable and we have

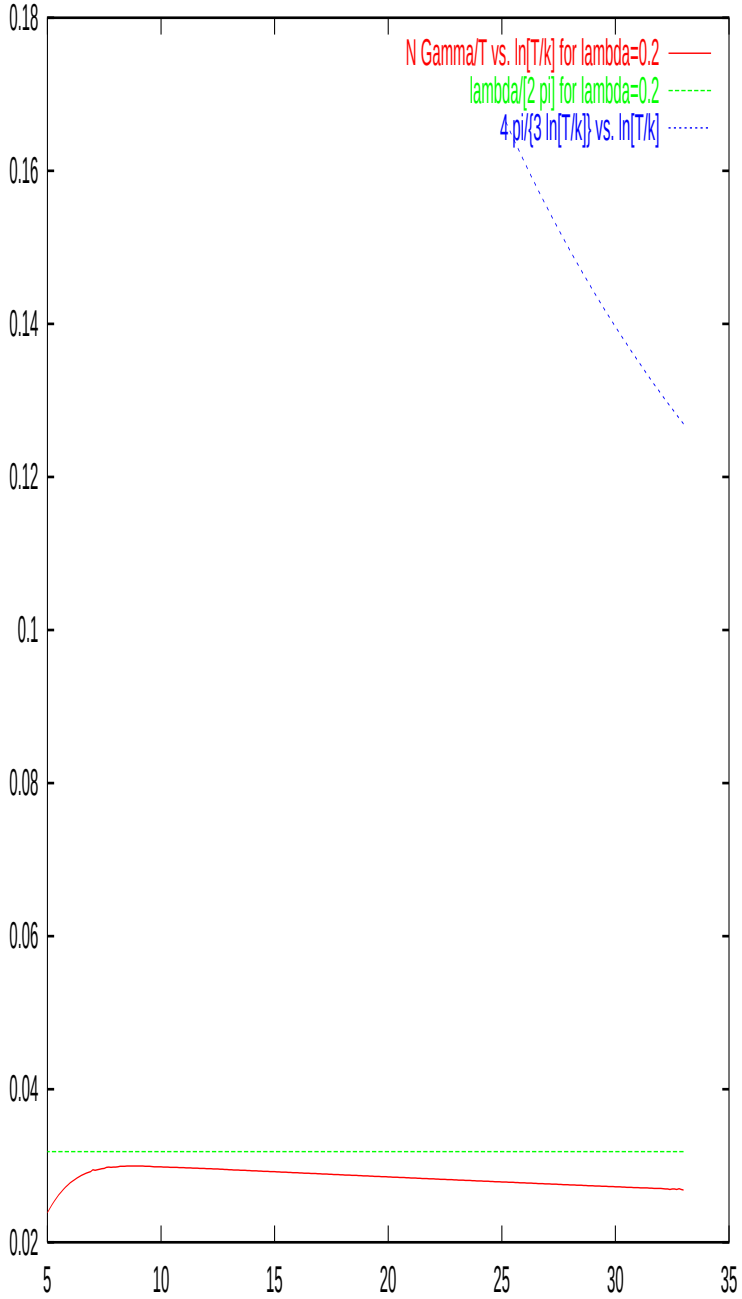
$$\Gamma_k(T) = \frac{\lambda T}{2N\pi} \left[1 + \mathcal{O}\left(\frac{1}{\ln \frac{\lambda T}{k}} \right) \right] \to const$$

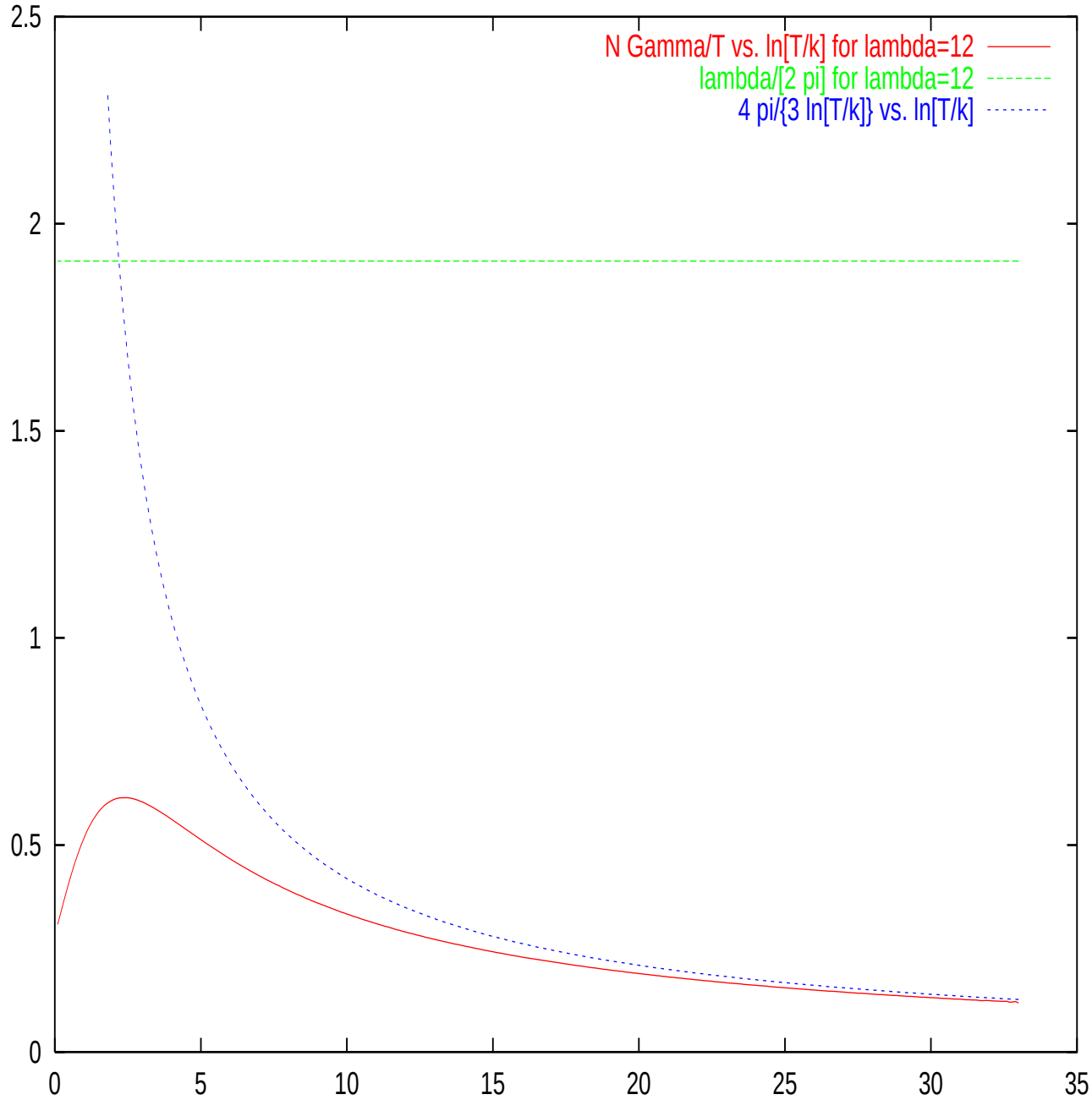
In regime 3 $(k < k_{us})$ large N resummation is needed and the classical approximation cannot be applied; only here we see the critical slowing down mechanism at work:

$$\Gamma_k(T) = \frac{4\,\pi\,T}{3\,N \ln \frac{T}{k}} \left[1 + \mathcal{O}\left(\frac{1}{\ln \frac{T}{k}} \right) \right] \to 0$$

In strong coupling $(\lambda/(4\pi) > 1)$ this is the only relevant regime.


N Gamma/T vs. ln[T/k] for lambda=1
lambda/[2 pi] for lambda=1
4 pi/{3 ln[T/k]} vs. ln[T/k]
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
5
10
15
20
25
30
35


N Gamma/T vs. ln[T/k] for lambda=0.2
lambda/[2 pi] for lambda=0.2
4 pi/{3 ln[T/k]} vs. ln[T/k]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
5
10
15
20
25
30
35



Strong coupling regime

Remarks

- At small coupling, critical slowing down can be seen only for extremely long wavelenghts:

$$\lambda = 0.2, \quad k < k_{us} = 0.82 \cdot 10^{-29}\, T$$

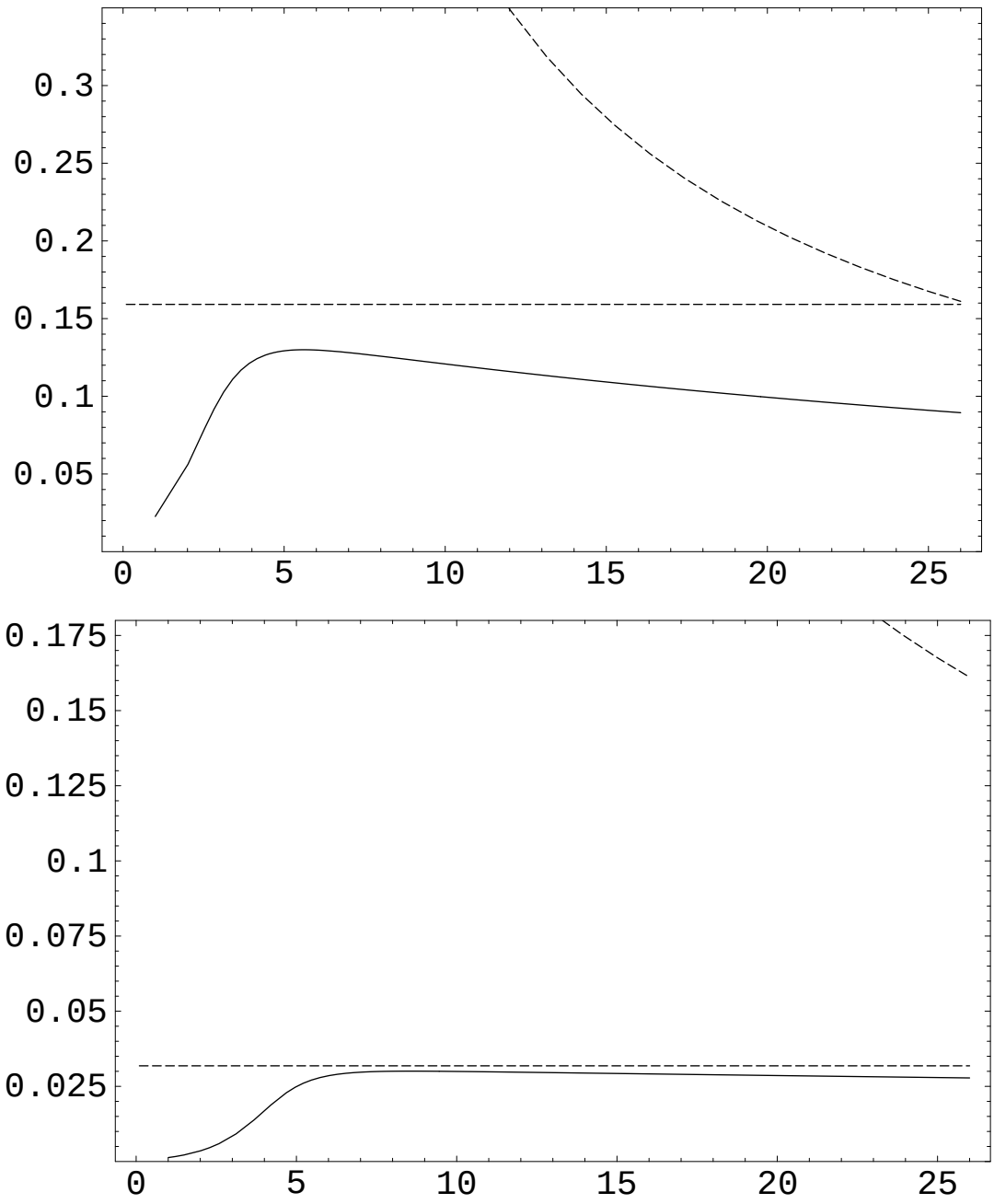
- At strong coupling the critical slowing down behavior appears immediately for momenta $k < T$,

$$\Gamma_k(T) = \frac{4\,\pi\,T}{3\,\ln\frac{T}{k}N}\left[1 + \mathcal{O}\left(\frac{1}{\ln\frac{T}{k}}\right)\right] + \frac{f(T/k)T}{N^2} + \cdots$$

  At very small k next-to-next-to-leading order corrections should be studied.

- In the near critical regime $m_T \ll T$ and for $k = 0$, the same results are obtained with $k \leftrightarrow m_T$.

Damping rate $\Gamma_0(T)/T, T > m_T$



Conclusions

1. At next-to-leading order in the $1/N$ expansion there is critical slowing down at $T = T_c$:

$$\Gamma_k(T) = \frac{4\,\pi\,T}{3\,N \ln \frac{T}{k}} \left[1 + \mathcal{O}\left(\frac{1}{\ln \frac{T}{k}} \right) \right] \qquad (k < k_{us})$$

However the relaxation time is only logarithmically increasing,

$$\tau_k(T) = \frac{3N}{4\pi} \frac{\ln(T/k)}{T} \;,\; k \sim k_{us} \;.$$

2. The dependence on λ is hidden in

$$k_{us}(\lambda, T) = \frac{\lambda T}{4\pi} \exp(-4\pi/\lambda)$$

and at small coupling $k_{us}(\lambda, T)$ is exponentially small, thus

$$\Gamma_k(T) \approx \frac{\lambda T}{2N\pi} = const$$

for many orders of magnitude.