GAUGE CONSISTENT WILSON RENORMALIZATION GROUP IN ALGEBRAIC NON-COVARIANT GAUGES

Michele SIMIONATO

LPTHE Jussieu PARIS & INFN

- Gauge Consistent Wilson Renormalization Group I & II, Int. J. Mod. Phys. A, Vol. 15, p. 2121 & 2153 (2000)

- On the Consistency of the Exact Renormalization Group Approach Applied to Gauge Theories in Algebraic Non-Covariant Gauges , hep-th/0005083, *to appear in* Int. J. Mod. Phys. A

Genova, November 29, 2000

(PARTIAL) LIST OF REFERENCES

- General

  Becchi (91)
  Bonini, D'Attanasio, Marchesini et Al. (93)
  Wetterich, Reuter et Al. (93)
  Ellwanger (94)
  Morris (98)
  Igarashi, Itoh, So (99)
  ...

- Noncovariant gauges

  Litim, Pawlowski (98)
  Geiger (99)
  Panza, Soldati (00).

- Others

  Blasi, Maggiore (96)
  Polonyi, Alexandre (99)

Plan of the seminar

I. THE MASS CUTOFF

II. NON-COVARIANT GAUGES

III. PROBLEMS OF THE $\Lambda \to 0$ LIMIT

IV. THE WILSON LOOP

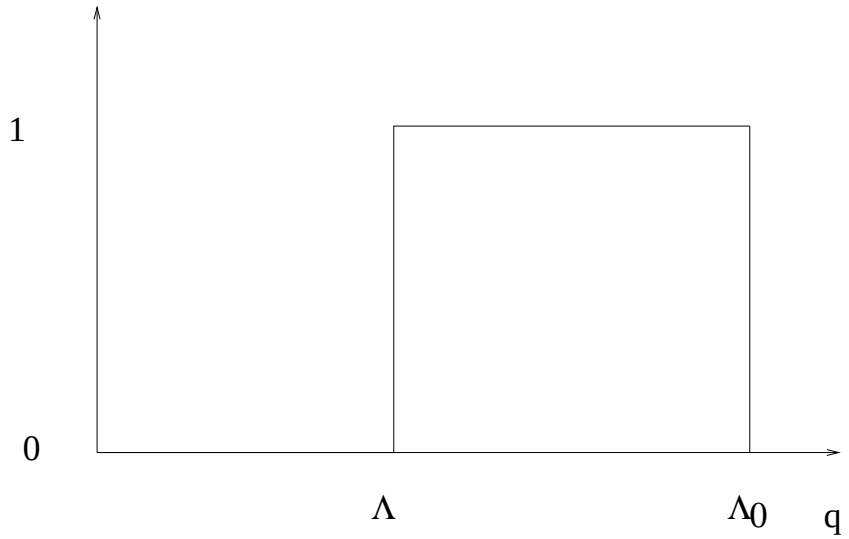
V. CONCLUSIONS

How to integrate UV degrees of freedom

Polchinski view:

$$\frac{1}{q^2} \to \frac{K_{\Lambda\Lambda_0}(q)}{q^2}$$

$$K_{\Lambda\Lambda_0}(q) = K_{\Lambda\infty}(q) - K_{\Lambda_0\infty}(q)$$



Wetterich view:

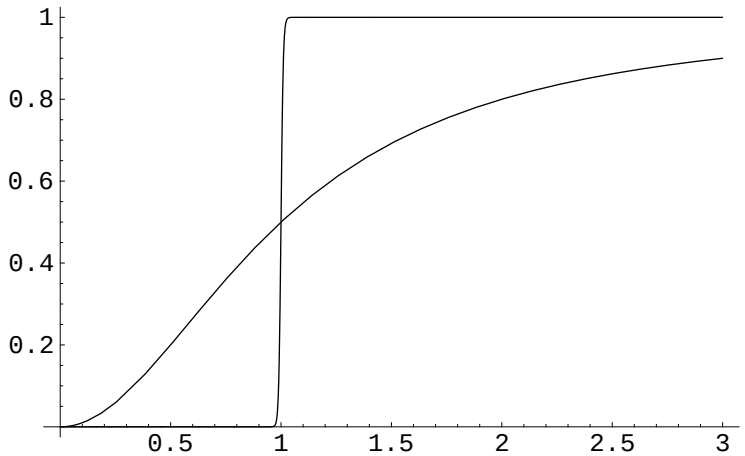
$$S_B(\phi) \to S_B(\phi) + \frac{1}{2}\int_q \phi(-q) R_\Lambda(q) \phi(q)$$

$$R_\Lambda(q) = q^2(K^{-1}_{\Lambda\infty}(q) - 1)$$

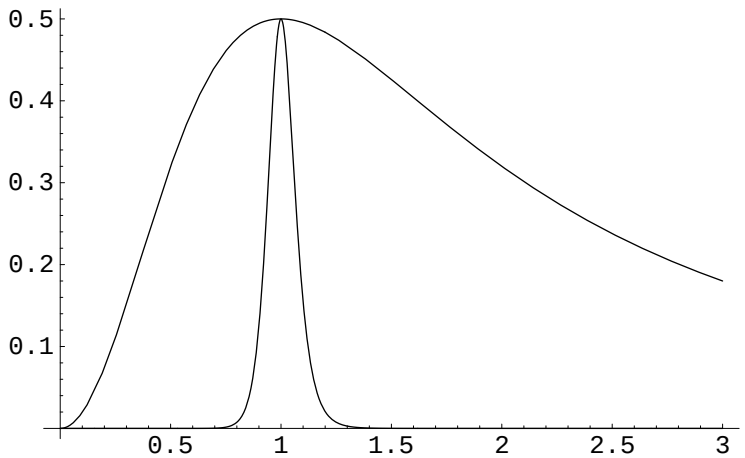
The mass cutoff

$$R_\Lambda = \Lambda^2, \qquad K_{\Lambda\infty}(q) = \frac{q^2}{q^2+\Lambda^2}$$

$K_{\Lambda\infty}(q)$



$-\Lambda\partial_\Lambda K_{\Lambda\infty}(q)$



UV problem of the mass cutoff

$$-\Lambda\partial_\Lambda K_{\Lambda\infty}(q) \sim \frac{\Lambda^2}{q^2}$$

$\Rightarrow$ quadratic divergences are not regularized.

Solution at the perturbative level

1. Introduce an explicit UV regularization (for instance higher derivatives).

2. Renormalize with suitable renormalization prescriptions (textbook exercise).

Remarks: i) the approach works even *out of perturbation theory* within suitable truncation schemes; ii) there are *no problems in 3D or in SUSY* theories without quadratic divergences.

Ward identities with a mass cutoff

QED:

$$\mathcal{W}_f \Gamma(\Phi, \Lambda) = \int_x \left(\frac{\Box}{\xi} + \Lambda^2 \right) \partial_\mu A^\mu f(x)$$

$$\mathcal{W}_f = - \int_x f(x) \left[\partial_\mu \frac{\delta}{\delta A_\mu} + ie\bar{\psi} \frac{\delta}{\delta \bar{\psi}} - ie\psi \frac{\delta}{\delta \psi} \right]$$

QCD:

analogous but only in axial or light-cone gauges.

The mass cutoff framework entails two problems:

1. gauge-dependence at $\Lambda \neq 0$;

2. singularity of the $\Lambda \to 0$ limit.

The gauge-fixing issue

$$S_B(\phi) = \int_x -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} i \not{D} \psi$$

gauge invariance for $\phi \to \phi + \delta_\omega \phi$

$\Longrightarrow$ no gauge field propagator !

The way out: gauge-fixing $\mathcal{F}(\phi) = 0$

$$Z_{\mathcal{F}} = \int [d\phi] \det \frac{\delta \mathcal{F}}{\delta \omega} \, \delta(\mathcal{F}(\phi)) e^{i S_B(\phi)}$$

$$< O(\phi) >_{\mathcal{F}} = \frac{1}{Z_{\mathcal{F}}} \int [d\phi] \det \frac{\delta \mathcal{F}}{\delta \omega} \, \delta(\mathcal{F}(\phi)) e^{i S_B(\phi)} O(\phi)$$

Gauge-independence of physical quantities

$$\delta_\omega O(\phi) = 0 \quad \Rightarrow \quad < O >_{\mathcal{F}} = < O >_{\mathcal{F}'}, \quad \mathcal{F} \neq \mathcal{F}'$$

BRST construction

$$Z_{\mathcal{F}} = \int [dC d\bar{C} d\lambda d\phi] \exp\left[iS_{\mathcal{F}}(\phi, C, \bar{C}, \lambda)\right]$$

$$S_{\mathcal{F}} = S_B + \lambda \cdot \mathcal{F}(A) - \bar{C} \cdot \frac{\partial \mathcal{F}}{\partial A_\mu} D_\mu C$$

$$sA_\mu = D_\mu C, \quad sC = -\frac{1}{2} g C \times C, \quad s\bar{C} = \lambda, \quad s\lambda = 0$$

Gauge-independence proof

$$\lambda \cdot \mathcal{F}_\varepsilon(A) - \bar{C} \cdot \frac{\delta \mathcal{F}_\varepsilon}{\delta A_\mu} D_\mu C = s\left(\bar{C} \cdot \mathcal{F}_\varepsilon(A)\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon} Z_\varepsilon = \int [d\Phi] e^{iS_{\mathcal{F}}(\Phi)} s\left(\bar{C} \cdot \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \mathcal{F}_\varepsilon(A)\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\varepsilon} < O >_\varepsilon = \frac{1}{Z_\varepsilon} \int [d\Phi] e^{iS_{\mathcal{F}}(\Phi)} s\left(O(\Phi)\, \bar{C} \cdot \frac{\mathrm{d}}{\mathrm{d}\varepsilon} \mathcal{F}_\varepsilon(A)\right)$$

$$= 0$$

This proof fails at $\Lambda \neq 0$!

Noncovariant linear gauges

$$\mathcal{F}(A) = L^\mu(p) A_\mu(p), \qquad L_\mu(p) = a n_\mu + b p_\mu$$

$$S^{gen}_{lin.g.} = \int_x -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \bar{C} \cdot n^\mu D_\mu C + \lambda \cdot n^\mu A_\mu$$

$$+\frac{\xi_2}{2} \lambda \cdot \lambda \,, \qquad \xi_2 \to 0$$

$$S^{red}_{lin.g.} = \int_x -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \bar{C} \cdot L^\mu D_\mu C + \int_x \frac{1}{2\xi_2} L^\mu A_\mu \cdot L^\nu A_\nu$$

$$S^{red}_{lin.g.}(A, C, \bar{C}, \Lambda) = S^{red}_{lin.g.} + \int_x \frac{1}{2} \Lambda^2 A^2$$

$$D^{-1}_{\Lambda,\mu\nu}(p_E) = \left(t_{\mu\nu}(p_E) p_E^2 + \frac{1}{\xi_2} L_\mu(p_E) L_\nu(p_E) + \Lambda^2 \delta_{\mu\nu} \right)$$

$$t_{\mu\nu}(p_E) = \delta_{\mu\nu} - \frac{p_{E,\mu} p_{E,\nu}}{p_E^2}$$

Linear gauge propagator

$$D_{\Lambda,\mu\nu}(p_E) \stackrel{\xi_2\to 0}{=} \frac{1}{p_E^2+\Lambda^2}\delta_{\mu\nu} -$$

$$\frac{p\cdot L(L_\mu p_{E,\nu} + L_\nu p_{E,\mu})}{(p_E^2+\Lambda^2)((p_E\cdot L)^2 + L^2\Lambda^2)} +$$

$$\frac{p_{E,\mu}p_{E,\nu}\ L^2}{(p_E^2+\Lambda^2)((p_E\cdot L)^2 + L^2\Lambda^2)} -$$

$$\frac{\Lambda^2 L_\mu L_\nu}{(p_E^2+\Lambda^2)((p_E\cdot L)^2 + L^2\Lambda^2)}.$$

$$L^\mu D_{\Lambda,\mu\nu}(p_E) \stackrel{\xi_2\to 0}{=} 0, \quad \forall\ \Lambda$$

Axial and light-cone gauges: $L^\mu \to n_E^\mu$

$n_E^2 > 0$ (axial) $n_E^2 = 0$ (light-cone)

Axial gauge propagator

$$n^\mu = (0,0,1,0)$$

$$D_{i3} = D_{3i} = D_{33} = 0 \ , \ i = 1,2,4 \ ,$$

$$D_{ij}(p) = \frac{1}{p^2+\Lambda^2}\left[\delta_{ij} + \frac{p_i p_j}{p_3^2+\Lambda^2}\right]$$

to be compared with the CPV propagator

$$D_{ij}(p) = \frac{1}{p^2}\left[\delta_{ij} + \frac{p_i p_j}{[p_3^2]}\right]$$

$$\frac{1}{[p_3^2]} = -\frac{\partial}{\partial p_3}\frac{1}{[p_3]} = \lim_{\Lambda\to 0}\frac{p_3^2-\Lambda^2}{(p_3^2+\Lambda^2)^2}$$

Problems of the pure axial gauge

• CPV prescription: many problems.

In particular the Euclidean propagator

$$D_E(p) \to \left(\frac{1}{p^2+\Lambda^2}, \frac{1}{p^2+\Lambda^2}, \frac{1}{[p_3^2]}, 0 \right)$$

is not positive definite $\Rightarrow$ the perturbative expansion of

$$Z(J) = \int [d\Phi] \exp\left[-\frac{1}{2}\Phi \cdot D_E^{-1}\Phi - V(\Phi) + J\Phi \right]$$

is doubtful.

• Wilsonian prescription: the Euclidean propagator is positive definite

$$D_E(p) \to \left(\frac{1}{p^2+\Lambda^2}, \frac{1}{p^2+\Lambda^2}, \frac{1}{p_3^2+\Lambda^2}, 0 \right)$$

and consistent with unitarity, but very singular at $\Lambda \to 0$ since

$$\frac{1}{[p_3^2]_W} = \lim_{\Lambda\to 0} \frac{1}{p_3^2+\Lambda^2} \to \frac{\pi}{\Lambda}\delta(p_3) \; ;$$

still problems should be expected.

The $x-$space propagator

$$D_{\Lambda,\mu\nu}(x) = \int_p e^{-ip\cdot x} D_{\Lambda,\mu\nu}(p)$$

$$D_{\Lambda,i3} = D_{\Lambda,3i} = D_{\Lambda,33} = 0$$

$$D_{\Lambda,ij}(x) \stackrel{\Lambda\to 0}{=} \frac{1}{2\Lambda}\int_{\bar{p}} \frac{p_i p_j}{\bar{p}^2} e^{-i\bar{p}\cdot\bar{x}}$$

$$= \frac{1}{2\Lambda}\frac{1}{3}\delta_{ij}\delta^{(3)}(\bar{x}) \stackrel{\Lambda\to 0}{=} \infty \; !$$

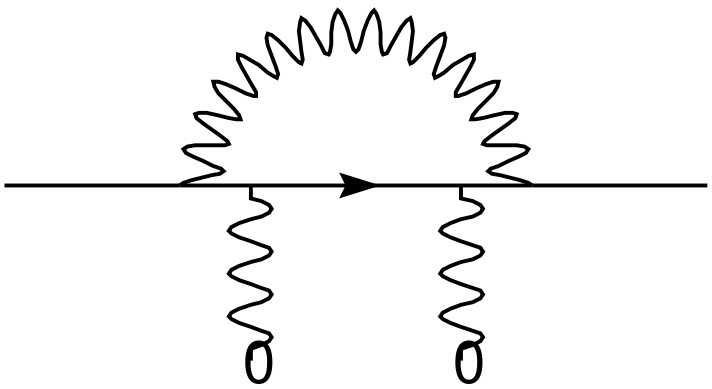
But the transverse part is regular ...

$$D^T_{\Lambda,\mu\nu}(x) \equiv \int_p e^{-ip\cdot x}\; t_{\mu\lambda}(p)\; D^{\lambda\rho}_{\Lambda}(p)\; t_{\rho\nu}(p)$$

$$\tilde{D}^T_{\Lambda,\mu\nu}(p_E) = \frac{\Lambda^2 t_{3\mu}(p_E) t_{3\nu}(p_E)}{(p_E^2+\Lambda^2)(p_3^2+\Lambda^2)}$$

$$\int_{p_E} \frac{\exp(ip_E\cdot x_E)}{p_E^2+\Lambda^2}\frac{\Lambda^2}{p_3^2+\Lambda^2} \stackrel{\Lambda\to 0}{=} O(\Lambda)$$

A one-loop computation



$$F(\bar{p}^2, p_3^2, \Lambda) = \int_{q_E} \left(\frac{\partial}{\partial m^2}\right)^2$$

$$\left[\frac{(2p_E + q_E)^\mu (2p_E + q_E)^\nu}{[(q_E + p_E)^2 + m^2 + \Lambda^2]} D_{\mu\nu}(q_E, \Lambda)\right]$$

Off-shell divergence of the axial gauge

$$F(\bar{p}^2, p_3^2, \Lambda) \overset{\Lambda \to 0}{=} -\frac{1}{2\Lambda} \int_{\vec{q}} \frac{2}{[(\bar{q} + \bar{p})^2 + m^2]^3} \times$$

$$q_i q_j (2p + q)^i (2p + q)^j \to \infty.$$

Euclidean light-cone propagator

$$D_{E,\mu\nu}(p_E) = \frac{1}{p_E^2 + \Lambda^2} \times$$

$$\left\{ \delta_{\mu\nu} - \frac{n_{E,\mu} p_{E,\nu} + n_{E,\nu} p_{E,\mu}}{p_E \cdot n_E} - \frac{\Lambda^2 n_{E,\mu} n_{E,\nu}}{(p_E \cdot n_E)^2} \right\}$$

Notations

$$n_E = (0, 0, 1, i), \quad n_E^* = (0, 0, 1, -i)$$

$$p_1 = p_\perp \cos\theta_\perp, \quad p_2 = p_\perp \sin\theta_\perp,$$

$$p_3 = p_\parallel \cos\theta_\parallel, \quad p_4 = p_\parallel \sin\theta_\parallel,$$

$$p_\parallel^2 = p_E \cdot n_E \; p_E \cdot n_E^* = p_3^2 + p_4^2,$$

$$\frac{1}{p_E \cdot n_E} = \frac{1}{i p_4 + p_3} = \frac{-i p_4 + p_3}{p_4^2 + p_3^2} = \frac{p_E \cdot n_E^*}{p_\parallel^2} = \frac{e^{-i\theta_\parallel}}{p_\parallel}$$

Continuation to Minkowski space

$$p_4^2 = -p_0^2 - i\varepsilon$$

$$-D_{\Lambda,\mu\nu}(p) = \frac{1}{p^2 - \Lambda^2 + i\varepsilon}\left\{g_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{[[p\cdot n]]} + \frac{\Lambda^2 n_\mu n_\nu}{[[p\cdot n]]^2}\right\}$$

$$\textbf{ML}: \quad \frac{1}{[[p\cdot n]]} \equiv \frac{p\cdot n^*}{(p\cdot n)(p\cdot n^*) + i\varepsilon},$$

$$n = (1,0,0,1), \quad n^* = (-1,0,0,1)$$

Ward identities

$$q^\mu D_{\mu\nu,\Lambda}(q) = \frac{n_\nu}{[[n\cdot q]]}\ ,$$

$$p^\mu D_\Lambda^{\lambda\nu}(q) V_{\nu\mu\rho}(q,p,-q-p) D_\Lambda^{\rho\tau}(q+p) = D_\Lambda^{\lambda\tau}(q) - D_\Lambda^{\lambda\tau}(q+p)\ ,$$

where

$$V_{\mu\nu\rho}(p,q,r) = (q-r)_\mu g_{\nu\rho} + (p-q)_\rho g_{\mu\nu} + (r-p)_\nu g_{\rho\mu}\ .$$

Light-cone propagator in $x-$space

$$D^{nn}_{\Lambda}(x_E) = -\Lambda^2 \int_{q_E} \frac{e^{iq_E\cdot x_E}}{(q_E\cdot n_E)^2(q_E^2+\Lambda^2)}$$

$$q_1 = q_\perp \cos\theta', \quad q_2 = q_\perp \sin\theta',$$

$$q_3 = q_\parallel \sin\theta, \quad q_4 = q_\parallel \cos\theta, \quad x = (x_\perp, 0, x_\parallel, 0)$$

$$D^{nn}_{\Lambda}(x_E) = -\Lambda^2 \int \frac{q_\perp dq_\perp d\theta'}{(2\pi)^2}\frac{dq_\parallel d\theta}{(2\pi)^2}\frac{e^{-2i\theta - iq_\parallel x_\parallel \sin\theta - iq_\perp x_\perp \sin\theta'}}{q_\parallel(q_\parallel^2+q_\perp^2+\Lambda^2)}$$

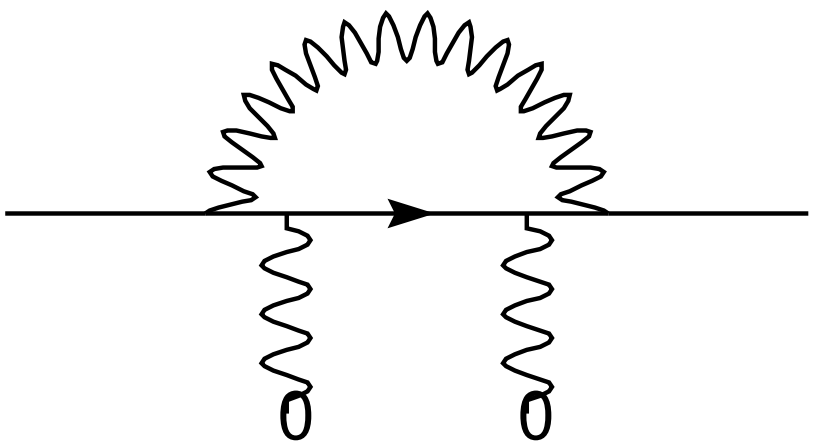
$$D^{nn}_{\Lambda}(x_E) = -\frac{\Lambda^2}{16\pi^2}\left[\frac{x_E^2}{x_\parallel^2}\log\frac{x_E^2}{x_\perp^2} - 1 + O(\Lambda x_\perp, \Lambda x_\parallel)\right]$$

The $\Lambda \to 0$ limit is regular, $O(\Lambda^2)$

A one-loop computation

$$D_{E,\mu\nu}(p_E) = D^{(a)}_{E,\mu\nu} + D^{(b)}_{E,\mu\nu} + D^{(c)}_{E,\mu\nu} =$$

$$\frac{\delta_{\mu\nu}}{p_E^2+\Lambda^2} - \frac{n_{E,\mu}p_{E,\nu} + n_{E,\nu}p_{E,\mu}}{(p_E\cdot n_E)(p_E^2+\Lambda^2)} - \frac{\Lambda^2 n_{E,\mu}n_{E,\nu}}{(p_E\cdot n_E)^2(p_E^2+\Lambda^2)}$$



$$F(p_\parallel^2, p_\perp^2, \Lambda) = F^{(a)} + F^{(b)} + F^{(c)}$$

is off-shell finite in the $\Lambda \to 0$ limit and consistent with the massless light-cone computation.

On-shell divergences

$$F^{(a)} = \int_q \left(\frac{\partial}{\partial m^2}\right)^2 \frac{(2p+q)^2}{(q^2+\Lambda^2)((q+p)^2+m^2+\Lambda^2)} \,,$$

$$F^{(b)} = \int_q \left(\frac{\partial}{\partial m^2}\right)^2 \frac{-2(2p+q)\cdot n\,(2p+q)\cdot q}{(q\cdot n)(q^2+\Lambda^2)((q+p)^2+m^2+\Lambda^2)},$$

$$F^{(c)} = \int_q \left(\frac{\partial}{\partial m^2}\right)^2 \frac{-\Lambda^2(2p\cdot n+q\cdot n)^2}{(q\cdot n)^2(q^2+\Lambda^2)((q+p)^2+m^2+\Lambda^2)}$$

$$F^{(a)}(p^2=-m^2,\Lambda) = -\frac{1}{16\pi}\frac{1}{m\Lambda} + O\left(\frac{\Lambda^0}{m^2}\right) \,,$$

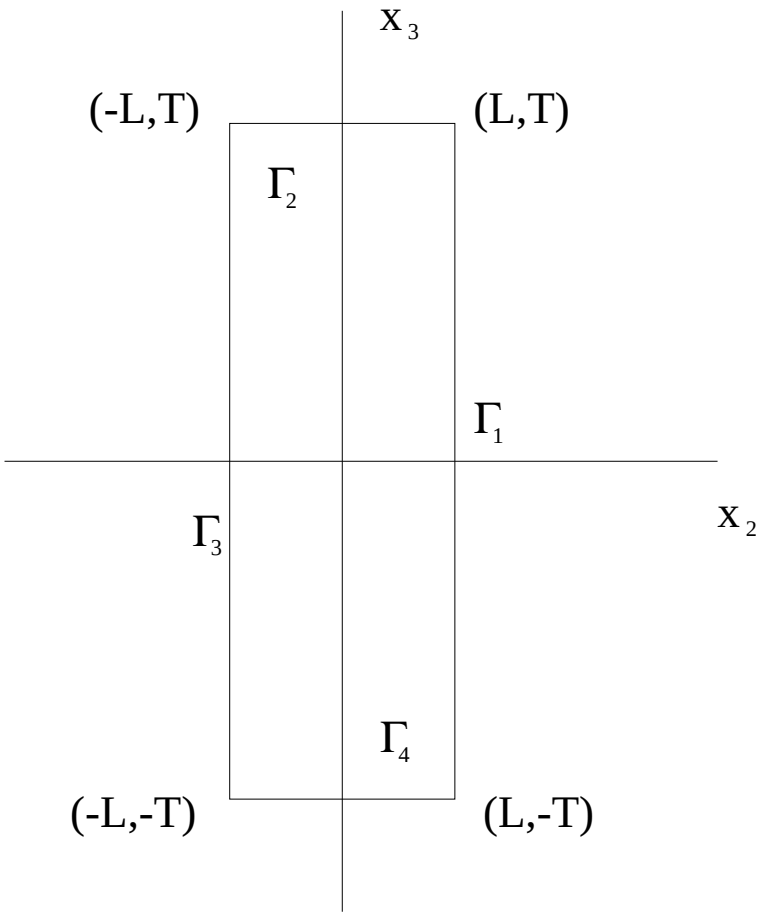
$$F^{(b)}(p_{\|}^2=-m^2,\Lambda) = \frac{1}{8\pi^2}\frac{1}{\Lambda^2} + O\left(\frac{\Lambda^0}{m^2}\right) \,,$$

$$F^{(c)}(p_{\|}^2=-m^2,\Lambda) = \frac{1}{6\pi^2}\frac{m^2}{\Lambda^4} + O\left(\frac{m^0}{\Lambda^2}\right) \,.$$

An analysis of gauge-invariant quantities is needed in order to really check the limit $\Lambda \to 0$.

The Wilson loop test

$$W_{\Gamma_{LT}} = \frac{1}{N_c} < \mathrm{Tr} P \exp\left(ig \int_{\Gamma_{LT}} A_\mu dx^\mu \right) >$$



$$\Gamma_{LT} = \Gamma^{(1)} + \Gamma^{(2)} + \Gamma^{(3)} + \Gamma^{(4)}$$

$$\Gamma^{(1)}(s) = sT \begin{pmatrix} 0 \\ 0 \\ +1 \\ 0 \end{pmatrix}, \quad \Gamma^{(2)}(s) = sL \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix},$$

$$\Gamma^{(3)}(s) = sT \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \quad \Gamma^{(4)}(s) = sL \begin{pmatrix} 0 \\ +1 \\ 0 \\ 0 \end{pmatrix},$$

$s \in [-1, 1]$.

$$W_{\Gamma_{LT}} = W^{(0)}_{\Gamma_{LT}} + g W^{(1)}_{\Gamma_{LT}} + \frac{g^2}{2} W^{(2)}_{\Gamma_{LT}} + \frac{g^3}{3!} W^{(3)}_{\Gamma_{LT}} + O(g^4)$$

$$= 1 - \frac{g^2}{4N_c} \int_{\Gamma_{LT}} dx^\mu dy^\nu < A_\mu(x) \cdot A_\nu(y) >^{(0)} + O(g^4)$$

$$W^{(2)}_{\Gamma_{LT}} = -C_F \int_{-1}^{1} ds_1 \int_{-1}^{1} ds_2 \frac{\mathrm{d}\Gamma^\mu}{\mathrm{d}s_1} \frac{\mathrm{d}\Gamma^\nu}{\mathrm{d}s_2} D_{\mu\nu}(\Gamma(s_1) - \Gamma(s_2))$$

Effective propagator in linear gauges

Since

$$\int_{\Gamma_{LT}} dx^\mu dy^\nu D_{\mu\nu}(x-y) = \int_{\Gamma_{LT}} dx^\mu dy^\nu D^T_{\mu\nu}(x-y)$$

the longitudinal terms of the propagator do not contribute to the $O(g^2)$ Wilson loop and we can decompose

$$D^{eff}_{\Lambda,\mu\nu} = \frac{\delta_{\mu\nu}}{p^2+\Lambda^2} - \frac{\Lambda^2 L_\mu L_\nu}{(p^2+\Lambda^2)((p\cdot L)^2 + L^2\Lambda^2)}$$

$$= \bar{D}_{\Lambda,\mu\nu} + \tilde{D}_{\Lambda,\mu\nu}$$

In the following we will take $L^\mu = n^\mu_E$, $n^2_E = 0$.

Interquark potential from the $\bar{D}_\Lambda$ term

$$V_\Lambda^{(2)}(2L) \equiv -\lim_{T\to\infty} \frac{g^2 N_c W^{(2)}_{\Gamma_{LT}}}{4T}$$

$$V_\Lambda^{(2)}(2L) = \frac{g^2 N_c C_F}{2T} \int_{-T}^{T} dt_1 \int_{-T}^{T} dt_2 \; \bar{D}_\Lambda(t_1+t_2, 2L)+c$$

Using

$$\lim_{T\to\infty} \frac{(2\sin p_3 T)^2}{p_3^2} = \lim_{T\to\infty} 2T \cdot 2\pi\delta(p_3)$$

follows

$$V_\Lambda^{(2)}(2L) = g^2 N_c C_F \int_{\bar{p}} \frac{\exp(ip_2 \cdot 2L)}{p^2+\Lambda^2}$$

$$= g^2 N_c C_F \frac{\exp(-\Lambda \cdot 2L)}{4\pi \cdot 2L}$$

Screened Coulomb potential

Relevance of the $\tilde{D}_\Lambda$ term

$$\tilde{V}^{(2)}_\Lambda(2L) = \frac{g^2 N_c C_F}{2T}\int_{-T}^{T} dt_1 \int_{-T}^{T} dt_2 D_{33}(0, 2L, t_1+t_2, 0)$$

$$= -\frac{g^2 N_c C_F}{2T}\int_{p_E} \frac{e^{ip_2\cdot 2L}\Lambda^2}{[p_3]^2(p_3^2+\bar{p}^2+\Lambda^2)}\frac{(2\sin p_3 T)^2}{p_3^2}\,.$$

Using the trick

$$\lim_{T\to\infty}\frac{(2\sin p_3 T)^2}{[p_3]^4} = \lim_{T\to\infty} -\frac{4}{3}T^3\cdot 2\pi\delta(p_3)$$

we obtain

$$\tilde{V}^{(2)}_\Lambda(2L) = \frac{2g^2 N_c C_F}{3}\Lambda^2 T^2\,\frac{\exp(-\Lambda\cdot 2L)}{4\pi\cdot 2L}\,.$$

In general

$$W_{\Gamma_{LT}} = g^2\cdot 2T\left[\bar{V}^{(2)}_\Lambda(2L) + c\Lambda^2\cdot(2T)^2\right] + O(g^4)$$

thus the Wilson loop test only works for loops of *finite size* $T << 1/\Lambda$.

Results

I. With the mass cutoff the RG flow preserves Ward Identities to all scales

II. If the $\Lambda \to 0$ limit exists for physical quantities, it is gauge independent

III. The pure axial gauge is ruled out

IV. The planar gauge is doubtful and moreover gauge symmetry is cumbersome

V. The light-cone gauge works, but only for finite Wilson loops

VI. In the computation of the interquark potential there is noncommutativity between the limits $\Lambda \to 0$ and $T \to \infty$.

Planar gauge

Massive action in Minkowski space

$$S_{planar}(\Lambda) = \int_x -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} - \bar{C} \cdot n^\mu D_\mu C + \lambda \cdot n^\mu A_\mu$$

$$+\frac{1}{2}\lambda \cdot \frac{n^2}{\partial^2 + \Lambda^2}\lambda + \frac{1}{2}\Lambda^2 A_\mu \cdot A^\mu \ ,$$

$$S^{red}_{planar}(\Lambda) = \int_x -\frac{1}{4} F_{\mu\nu} \cdot F^{\mu\nu} + \frac{1}{2}\Lambda^2 A \cdot A +$$

$$-\frac{1}{2n^2} n_\mu A^\mu \cdot (\partial^2 + \Lambda^2) n_\mu A^\mu - \bar{C} \cdot n^\mu D_\mu C \ .$$

Mass term on transverse degrees of freedom

$$\frac{1}{2} A^\mu \left(g_{\mu\nu} - \frac{n_\mu n_\nu}{n^2} \right) A^\nu = \frac{1}{2} A^i g_{ij} A^j, \quad i, j \in \{0, 1, 2\}$$

Planar gauge propagator

$$-D_{\Lambda,\mu\nu}(p) = \frac{1}{p^2 - \Lambda^2 + i\varepsilon} \times$$

$$\left\{ g_{\mu\nu} - \frac{n_\mu p_\nu + n_\nu p_\mu}{[p \cdot n]} + \frac{\Lambda^2 n_\mu n_\nu}{[p \cdot n]^2} \right\}$$

1. Spurious divergences need a prescription

$$\frac{1}{[p \cdot n]} \equiv \lim_{\varepsilon \to 0} \frac{p \cdot n}{(p \cdot n)^2 + \varepsilon^2}$$

2. Natural decomposition

$$D_{\Lambda,\mu\nu} = \bar{D}_{\Lambda,\mu\nu} + \tilde{D}_{\Lambda,\mu\nu}$$

$$\tilde{D}_{\Lambda,\mu\nu} = -\frac{1}{p^2 - \Lambda^2 + i\varepsilon} \times \frac{\Lambda^2 n_\mu n_\nu}{[p \cdot n]^2}$$

The splitting formula

$$\frac{1}{[nk]}\frac{1}{[n(p-k)]} = \frac{1}{np}\left(\frac{1}{[nk]} + \frac{1}{[n(p-k)]}\right) +$$

$$-\pi^2\delta(nk)\delta(np)$$

$$\frac{1}{[x]} = \lim_{\varepsilon\to 0}\ \frac{x}{x^2+\varepsilon^2}, \qquad \delta(x) = \lim_{\varepsilon\to 0}\ \frac{\varepsilon/\pi}{x^2+\varepsilon^2}$$

In two-loops (and higher orders) diagrams in non-abelian gauge theories there are contributions involving the *ill-defined* object

$$\lim_{\varepsilon\to 0}\ \delta(x)\frac{1}{[x]} = \infty\ .$$

Thus the perturbative expansion in axial (and planar) gauge is *extremely* delicate.

Gauge-dependence

At $\Lambda \neq 0$ there is gauge-dependence on the vector n^μ (i.e. Lorentz-covariance breaking) constrained by an extended ST-identity:

$$-\varphi_\mu \frac{\partial \Gamma}{\partial n_\mu} = \int_x \left(\frac{\delta \Gamma}{\delta A^*} \cdot \frac{\delta \Gamma}{\delta A} + \frac{\delta \Gamma}{\delta C^*} \cdot \frac{\delta \Gamma}{\delta C} + \lambda \cdot \frac{\delta \Gamma}{\delta \bar{C}} \right) + \Lambda^2 \frac{\partial \Gamma}{\partial \bar{\rho}}$$

where there is decoupling of ghosts and auxiliary fields:

$$\frac{\delta \Gamma}{\delta \bar{C}} - gC \times \frac{\delta \Gamma}{\delta \lambda} = -n^\mu \partial_\mu C - \varphi^\mu A_\mu$$

$$\frac{\delta \Gamma}{\delta C} - g\bar{C} \times \frac{\delta \Gamma}{\delta \lambda} = -n^\mu \partial_\mu \bar{C} - \bar{\rho} \partial_\mu A^\mu + \partial^\mu A^*_\mu - gA^* \times A - gC^* \times C$$

$$\frac{\delta \Gamma}{\delta \lambda} = n^\mu A_\mu$$