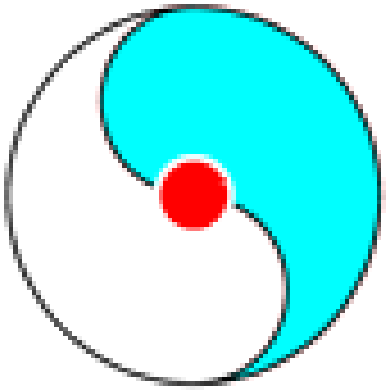
OUT OF EQUILIBRIUM NON-PERTURBATIVE PHOTOPRODUCTION

Michele SIMIONATO
LPTHE Jussieu PARIS



QUANTUM FIELDS IN & OUT OF EQUILIBRIUM

BNL, Brookhaven, N.Y., USA
October 25-30 1998

Work in collaboration with

D. BOYANOVSKY

H. J. de VEGA

R. HOLMAN

References

- Analytic and Numerical study of preheating dynamics
  D. Boyanovsky, H. J. de Vega, R. Holman and J. F. J. Salgado, Physical Review D **54**, 7570-7598 (1996),

- Photoproduction Enhancement From Non Equilibrium Disoriented Chiral Condensates
  D. Boyanovsky, H. J. de Vega, R. Holman and S. P. Kumar, Physical Review D **56**, 5233-5250 (1997)

- Non-Equilibrium Production of Photons via $\pi^0 \to 2\gamma$ in DCC's, D. Boyanovsky, H. J. De Vega, R. Holman, S. P. Kumar, Physical Review D **56**, 3929 (1997)

Plan of the seminar

I. THE MODEL

II. ANALYTICAL ESTIMATES

III. PHOTOPRODUCTION

IV. MAGNETIC MASS

V. CONCLUSIONS

I. THE MODEL

Field contents:

$$(\sigma, \Phi_r, \Phi_r^\dagger, A_T^i, A_0), \quad r = 1, \ldots, N, \quad i = 1, 2, 3$$

$$\nabla \cdot \vec{A}_T = 0$$

Lagrangian:

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$$

$$\mathcal{L}_1 = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\Phi^\dagger\partial^\mu\Phi - m^2\left(\frac{1}{2}\sigma^2 + \Phi^\dagger\Phi\right) +$$

$$-\frac{\lambda}{2N}\left(\frac{1}{2}\sigma^2 + \Phi^\dagger\Phi\right)^2,$$

$$\mathcal{L}_2 = \frac{1}{2}\partial_\mu\vec{A}_T \cdot \partial^\mu\vec{A}_T + \frac{1}{2}(\nabla A_0)^2,$$

$$\mathcal{L}_3 = -\frac{ie}{\sqrt{N}}\vec{A}_T \cdot \left(\Phi^\dagger\nabla\Phi - \nabla\Phi^\dagger\Phi\right) +$$

$$-\frac{e^2}{N}\left(\vec{A}_T^2 - A_0^2\right)\Phi^\dagger\Phi - \frac{ie}{\sqrt{N}}A_0\left(\Phi\dot{\Phi}^\dagger - \Phi^\dagger\dot{\Phi}\right)$$

Quantum equations of motion

$$
\begin{aligned}
(\Box + m^2) < \sigma > &= -\frac{\lambda}{N} < \left(\frac{1}{2}\sigma^2 + \Phi^\dagger\Phi\right)\Phi > \\
(\Box + m^2) < \Phi > &= -\frac{\lambda}{N} < \left(\frac{1}{2}\sigma^2 + \Phi^\dagger\Phi\right)\Phi > + e.m. \\
(\Box + m^2) < \Phi^\dagger > &= -\frac{\lambda}{N} < \left(\frac{1}{2}\sigma^2 + \Phi^\dagger\Phi\right)\Phi^\dagger > + e.m. \\
\Box < A_T > &= < \vec{J}(\Phi^\dagger, \Phi, A_T) > \\
\nabla^2 < A_0 > &= < \rho(\Phi^\dagger, \Phi, A_0) >
\end{aligned}
$$

Analyzed regime:

$$
\begin{aligned}
< \sigma > &= \sqrt{N}\phi(t) \\
< \Phi > &= < \Phi^\dagger > = 0 \\
< A_T > \quad , \quad &< A_0 > \ \ small
\end{aligned}
$$

$$
\phi(t_0) \sim \frac{M_R}{\sqrt{\lambda}} \ large
$$

LARGE N APPROXIMATION

Equations of condensates

$$\left(\Box + m^2 + \frac{\lambda}{2} < \psi^2 > + \frac{\lambda}{2}\phi^2\right)\phi = 0$$

$$(\Box + e^2 < \psi^2 >) A_T = \frac{-e}{\sqrt{N}} < \Phi^\dagger i\nabla\Phi - i\nabla\Phi^\dagger\Phi >$$

$$(-\nabla^2 + e^2 < \psi^2 >) A_0 = \frac{-e}{\sqrt{N}} < \Phi^\dagger i\partial_t\Phi - i\partial_t\Phi^\dagger\Phi >$$

Equation of fluctuations

$$\left(\Box + m^2 + \frac{\lambda}{2} < \psi^2 > + \frac{\lambda}{2}\phi^2\right) < \psi^2 > = 0$$

Definitions

$$\frac{1}{2} < \psi(t)^2 > \equiv \frac{1}{N}\sum_{r=1}^{N} < \Phi_r^\dagger \Phi_r >$$

$$\psi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2}}\left[a_k\varphi_k(t)e^{i\vec{k}\cdot\vec{x}} + a_k^\dagger\varphi_k^*(t)e^{-i\vec{k}\cdot\vec{x}}\right]$$

Renormalization

$$< \psi(t)^2 >= \int \frac{d^3k}{(2\pi)^3\sqrt{2}} |\varphi_k(t)|^2$$

$$m_R^2 = m^2 + \frac{\lambda}{16\pi^2}\left(\Lambda^2 - m_R^2 \log\frac{\Lambda}{\kappa}\right)$$

$$\lambda_R = \lambda\left(1 - \frac{\lambda_R}{16\pi^2}\log\frac{\Lambda}{\kappa}\right)$$

$$< \psi^2(t) >_R = \int \frac{k^2\,dk}{4\pi^2}\left\{|\varphi_q(t)|^2 - \frac{1}{k} + \right.$$

$$\left.\frac{\Theta(k-\kappa)}{2k^3}\left[m_R^2 + \frac{\lambda_R}{2}\phi(t)^2 + \frac{\lambda_R}{2} < \psi(t)^2 >_R\right]\right\}$$

$$M_R^2 = m_R^2 + \frac{\lambda_R}{2} < \psi^2(0) >_R$$

Definitions

Dimensionless variables

$$\tau = |M_R| t, \;\; q = k/|M_R|, \;\; g = \frac{\lambda_R}{8\pi^2},$$

$$\eta^2(t) = \frac{\lambda_R}{2|M_R^2|}\phi^2(t), \;\; \kappa = |M_R|,$$

Backreaction

$$\begin{aligned} g\Sigma_R(\tau) &= \frac{\lambda_R}{2|M_R^2|}\left(< \psi(t)^2 >_R - < \psi(0)^2 >_R\right) \\ &= g\int q^2\, dq \Bigg\{ |\varphi_q(\tau)|^2 - |\varphi_q(0)|^2 \\ &\quad + \frac{\Theta(q-1)}{2q^3}\left[-\eta_0^2 + \eta(\tau)^2 + g\Sigma(\tau)\right]\Bigg\} \end{aligned}$$

Large N scalar equations

Renormalized equations of motion:

$$\left[\frac{d^2}{d\tau^2} \quad \pm 1 + \eta(\tau)^2 + g\Sigma(\tau)\right]\eta(\tau) \quad = 0$$

$$\left[\frac{d^2}{d\tau^2} + q^2 \quad \pm 1 + \eta(\tau)^2 + g\Sigma(\tau)\right]\varphi_q(\tau) \quad = 0$$

Initial values:

$$\eta(0) = \eta_0, \qquad \dot{\eta}(0) = 0$$

$$\varphi_q(0) = \frac{1}{\sqrt{\Omega_q}}, \qquad \dot{\varphi}_q(0) = -i\sqrt{\Omega_q}$$

$$\Omega_q = \sqrt{q^2 + 1 + \eta_0^2}$$

II. ANALYTICAL ESTIMATES

Early time regime

Initially the fluctuations are small ($\Sigma(0) = 0$) and can be neglected:

$$\ddot{\eta} \pm \eta + \eta^3 = 0$$

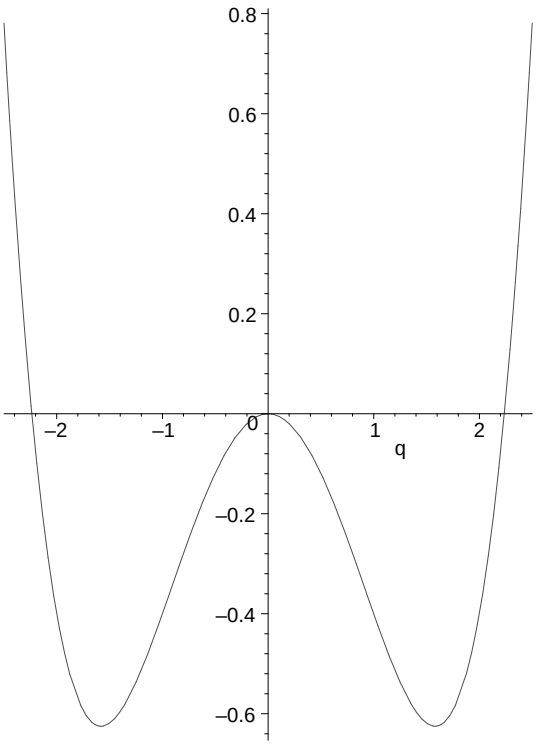
$$\left[\frac{d^2}{d\tau^2} + q^2 \pm 1 + \eta^2(\tau)\right]\varphi_q(\tau) = 0$$

The evolution of the order parameter and the evolution of the modes are decoupled and can be analytically analyzed. Then one compute back the fluctuations: they exponentially increases.

- Broken symmetry: spinodal instability

- Unbroken symmetry: parametric resonance

BROKEN PHASE



Early time mode functions

$$\varphi_q(\tau) = \alpha_q \exp\left(\tau\sqrt{1-q^2}\right) + \alpha_q^* \exp\left(-\tau\sqrt{1-q^2}\right),$$

$$\alpha_q = \frac{\sqrt{1-q^2} - i\sqrt{1+q^2}}{2\sqrt{1-q^2}(1+q^2)^{1/4}}.$$

Broken phase

Figure 4 a

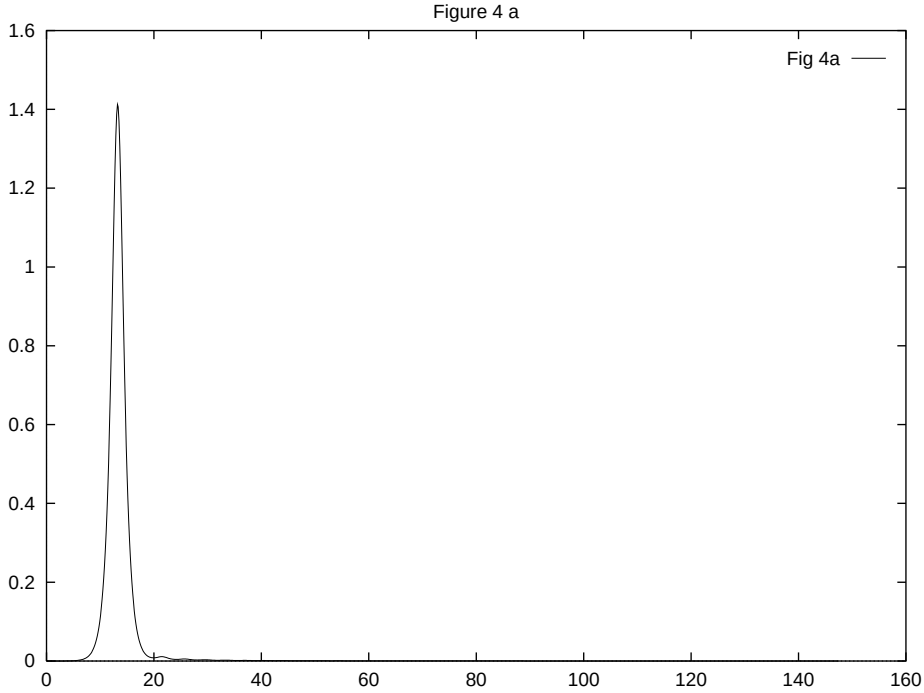
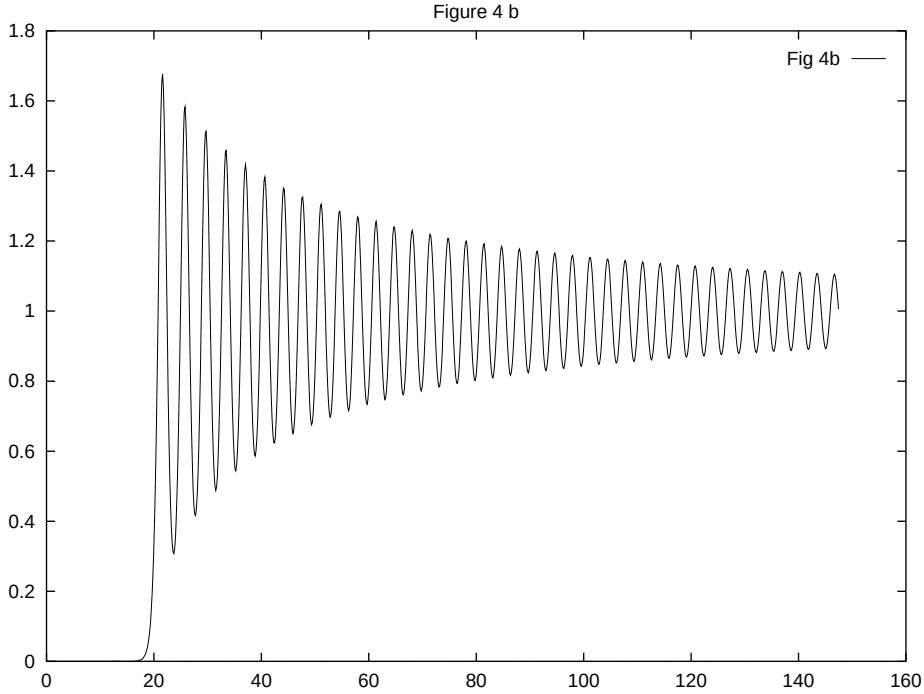
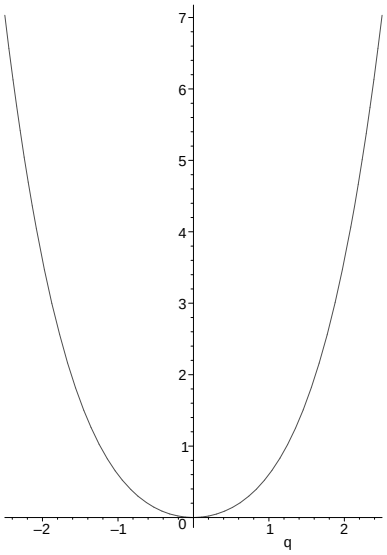


Figure 4 b



Unbroken phase



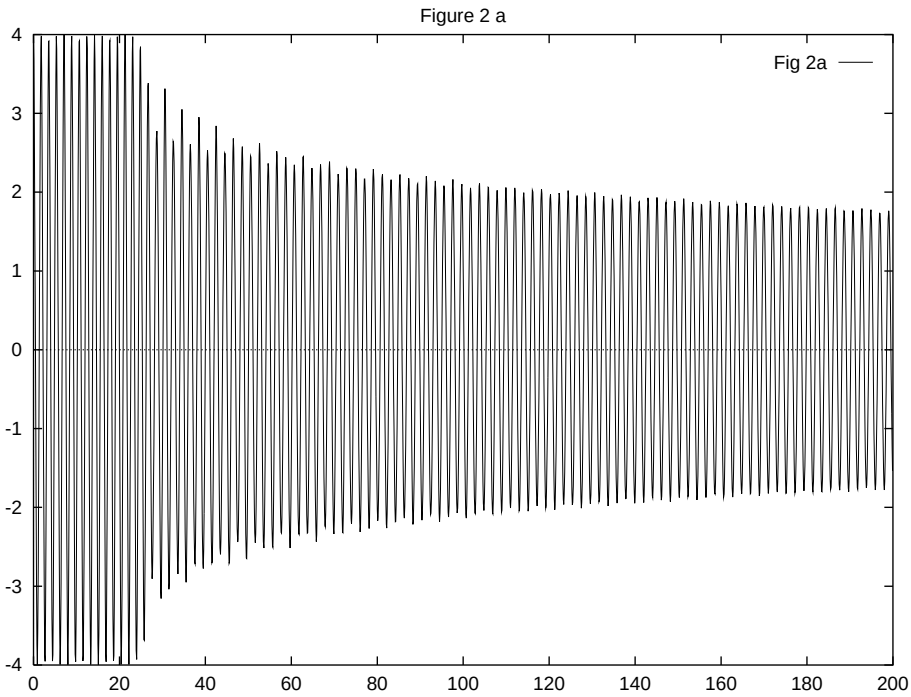
Early time mode functions

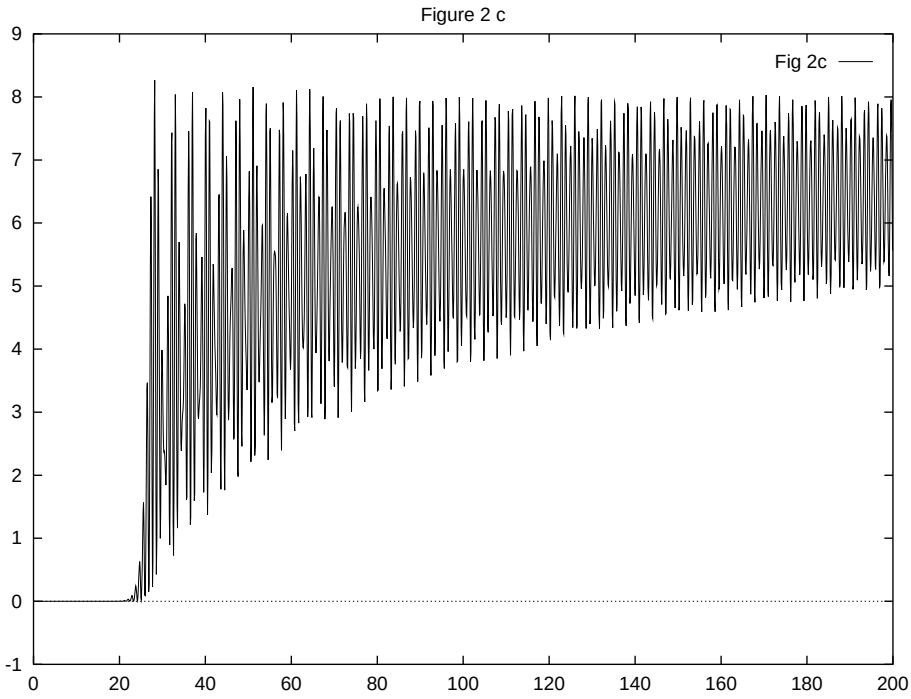
$$\varphi_q(\tau) = \alpha_q \; U_q(-\tau) + \alpha_q^* \; U_q(\tau)$$

$$\alpha_q = \frac{1}{2\sqrt{\Omega_q}} \left(1 - \frac{2i\Omega_q}{W_q} \right),$$

$$\Omega_q = \sqrt{q^2 + 1 + \eta_0^2}, \qquad W_q = -2q \sqrt{\frac{\eta_0^2/2 + 1 + q^2}{\eta_0^2/2 - q^2}}.$$

Unbroken phase





Floquet functions

$$\left[\frac{d^2}{d\tau^2} + q^2 + 1 + \eta^2(\tau)\right]\varphi_q(\tau) = 0$$

$$\eta^2(\tau + 2\omega) = \eta^2(\tau), \quad 2\omega(\eta_0) \simeq \frac{\pi}{\sqrt{1+\eta_0^2}}$$

$$U_q(\tau + 2\omega) = e^{iF_q} U_q(\tau)$$

Bands $0 \le q^2 \le \eta_0^2/2$ and $q^2 > \eta_0^2/2$

$$U_q(-\tau) \simeq e^{B_q \tau} \frac{\sin\left(\pi v_q - \sqrt{1+\eta_0^2}\,\tau\right)}{\sin \pi v_q},$$

$$B_q(\eta_0) = 4\sqrt{1+\eta_0^2}\;\widehat{q}(\eta_0)\sin 2\pi v_q$$

$$\sin \pi v_q = \sqrt{1 - \frac{2}{\eta_0^2}q^2}, \quad \cos \pi v_q = \frac{\sqrt{2}}{\eta_0}q$$

$$\widehat{q}(\eta_0) < e^{-\pi} = 0.0432139... \quad \forall \eta_0$$

NON-EQUILIBRIUM PROPAGATORS

•Scalar Propagators (leading order in $1/N$)

$$\langle {\Phi_r}^{(a)\dagger}(\vec{x},t){\Phi_s}^{(b)}(\vec{x'},t')\rangle =$$

$$-i\delta_{rs}\int \frac{d^3k}{(2\pi)^3}\, G_k^{ab}(t,t')\, e^{-i\vec{k}\cdot(\vec{x}-\vec{x'})}$$

where $(a,b) \in \{+,-\}$.

$$G_k^{++}(t,t') = G_k^{>}(t,t')\Theta(t-t') + G_k^{<}(t,t')\Theta(t'-t)$$
$$G_k^{--}(t,t') = G_k^{>}(t,t')\Theta(t'-t) + G_k^{<}(t,t')\Theta(t-t')$$
$$G_k^{\pm\mp}(t,t') = -G_k^{<(>)}(t,t')$$
$$G_k^{>}(t,t') = i\varphi_k(t)\varphi_k^*(t')$$
$$G_k^{<}(t,t') = i\varphi_k(t')\varphi_k^*(t)$$

- Photon Propagator (tree level)

$$\langle A_{Ti}^{(a)}(\vec{x},t) A_{Tj}^{(b)}(\vec{x'},t')\rangle =$$

$$-i \int \frac{d^3k}{(2\pi)^3}\, \mathcal{P}_{ij}(\vec{k})\; \mathcal{G}_k^{ab}(t,t')\; e^{-i\vec{k}\cdot(\vec{x}-\vec{x'})}$$

$$\mathcal{P}_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2}\ ,$$

$$\mathcal{G}_k^{++}(t,t') = \mathcal{G}_k^{>}(t,t')\Theta(t-t') + \mathcal{G}_k^{<}(t,t')\Theta(t'-t)$$
$$\mathcal{G}_k^{--}(t,t') = \mathcal{G}_k^{>}(t,t')\Theta(t'-t) + \mathcal{G}_k^{<}(t,t')\Theta(t-t')$$
$$\mathcal{G}_k^{\pm\mp}(t,t') = -\mathcal{G}_k^{<(>)}(t,t')$$
$$\mathcal{G}_k^{>}(t,t') = \frac{i}{2k}\, e^{-ik(t-t')}$$
$$\mathcal{G}_k^{<}(t,t') = \frac{i}{2k}\, e^{ik(t-t')}$$

III. PHOTOPRODUCTION

Photon number:

$$N_{ph}(k,t) \equiv < a_T^{\dagger}(k,t) \cdot a_T(k,t) > =$$

$$\frac{1}{2\omega_k} \left\langle \dot{A}_T(\vec{k},t) \cdot \dot{A}_T(-\vec{k},t) + \omega_k^2 A_T(\vec{k},t) \cdot A_T(-\vec{k},t) \right\rangle$$

Production rate:

$$\dot{N}(k,t) \equiv \dot{N}_{ph}(k,t)/V =$$

$$\frac{-i}{2\omega_k} \frac{\partial}{\partial t_2} \left[\left(\frac{\partial^2}{\partial t_2^2} + \omega_k^2 \right) \left(\mathcal{G}^{>}_{k,ii}(t_1,t_2) + \mathcal{G}^{<}_{k,ii}(t_1,t_2) \right) \right]_{t_1=t_2=t}$$

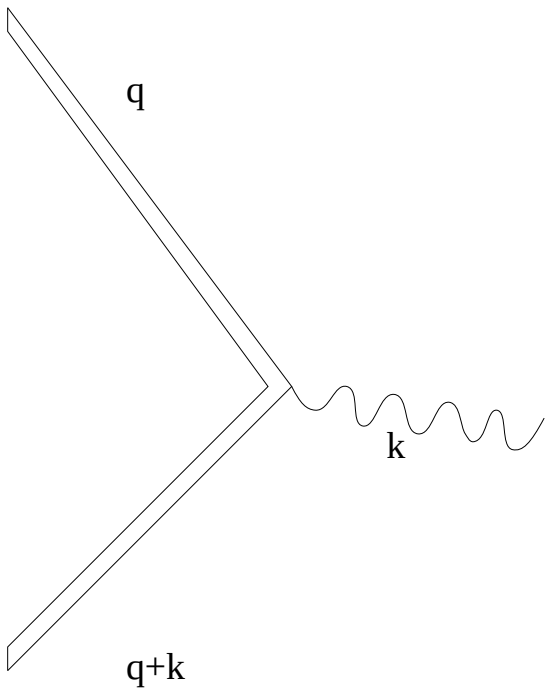
$$= -\frac{i}{\omega_k} \int_{t_0}^{t} \int \frac{d^3q}{(2\pi)^3} \left[\Pi^{>}_{q+k,ij}(t,t') \partial_t \mathcal{G}^{<}_{q,ji}(t',t) \right.$$

$$\left. - \Pi^{<}_{q+k,ij}(t,t') \partial_t \mathcal{G}^{>}_{q,ji}(t',t) \right] dt'.$$

At first order in e^2 and $1/N$

$$N_{ph}(k,t) = \frac{2e^2}{k}\int \frac{d^3q}{(2\pi)^3}\, q^2\,(1-\cos^2\theta)\cdot \left|\int_{t_0}^{t}\varphi_q(t_1)\varphi_{q+k}(t_1)e^{-ikt_1}dt_1\right|^2 .$$

Corresponds to the off-shell process $\Phi \rightarrow \Phi + \gamma$



Analytic results

- Broken phase

$$N_{ph}(k,t) = N_1 \frac{|M_R|^{1/2} e^2}{k^3 t^{5/2}}\, e^{t[4-k^2/2+O(k^4)]} + O(1/t)$$

$$N_1 = \frac{1}{256\sqrt{2\pi^3}}$$

- Unbroken phase

$$N_{ph}(k,t) = N_2 \frac{|M_R| e^2}{k^2 t}\, e^{t\left[B(\eta_0)+O(k^3)\right]} + O(1/t)$$

$$B(\eta_0) = 16\ \sqrt{1+\eta_0^2}\ \hat{q}(\eta_0) \sim 0.69\ \eta_0$$

$$N_2 = \frac{1}{1024\pi} \frac{\sqrt{2}\ \eta_0^2}{\sqrt{5\ \eta_0^2+4}(3\ \eta_0^2+4)\ \hat{q}^3(\eta_0)\sqrt{1+\eta_0^2}}.$$

IV. MAGNETIC MASS IN THE ASYMPTOTIC REGIME

At very large t

$$G_>(t,p) = \frac{i}{2\omega_p}\left[e^{i\omega_p t}\tilde{n}(\omega_p) + e^{-i\omega_p t}(\tilde{n}(\omega_p)+1)\right]$$

where $\tilde{n}(\omega_p)$ is non-thermal.

From linear response theory

$$\frac{d^2}{dt^2}\mathcal{A}_{Ti}(\vec{k},t) + k^2\mathcal{A}_{Ti}(\vec{k},t) + e^2 < \psi^2 > \mathcal{A}_{Ti}(\vec{k},t)$$
$$-2e^2\int_0^t d\tau \int \frac{d^3p}{(2\pi)^3\omega_p\omega_{p+k}}\, p_{Ti}\, p_{Tj}\, [(1+\tilde{n}_p+$$
$$\tilde{n}_{p+k})\sin\{(\omega_{k+p}+\omega_p)(t-\tau)\} + (\tilde{n}_p - \tilde{n}_{k+p})\cdot$$
$$\sin\{(\omega_{k+p}-\omega_p)(t-\tau)\}]\,\mathcal{A}_{Tj}(\vec{k},\tau) = \mathcal{J}_{Ti}(\vec{k},t)$$

Asymptotic magnetic mass

Laplace transform

$$(s^2 + k^2)\, \tilde{\mathcal{A}}_{Ti}(\vec{k}, s) + \Sigma_{ij}(\vec{k}, s)\, \tilde{\mathcal{A}}_{Tj}(\vec{k}, s)$$

$$= s\mathcal{A}_{Ti}(\vec{k}, 0) + \dot{\mathcal{A}}_{Ti}(\vec{k}, 0) + \tilde{\mathcal{J}}_{Ti}(\vec{k}, s)$$

Results

I. We have non-equilibrium non-perturbative predictions in a solid computational framework

II. We prove the asymptotic vanishing of the magnetic mass even for non-thermal distribution

Physical applications

I. Enhanced protoproduction in chiral phase transition

II. Large magnetic fields in cosmological phase transition