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THE MESON SPECTRUM

by

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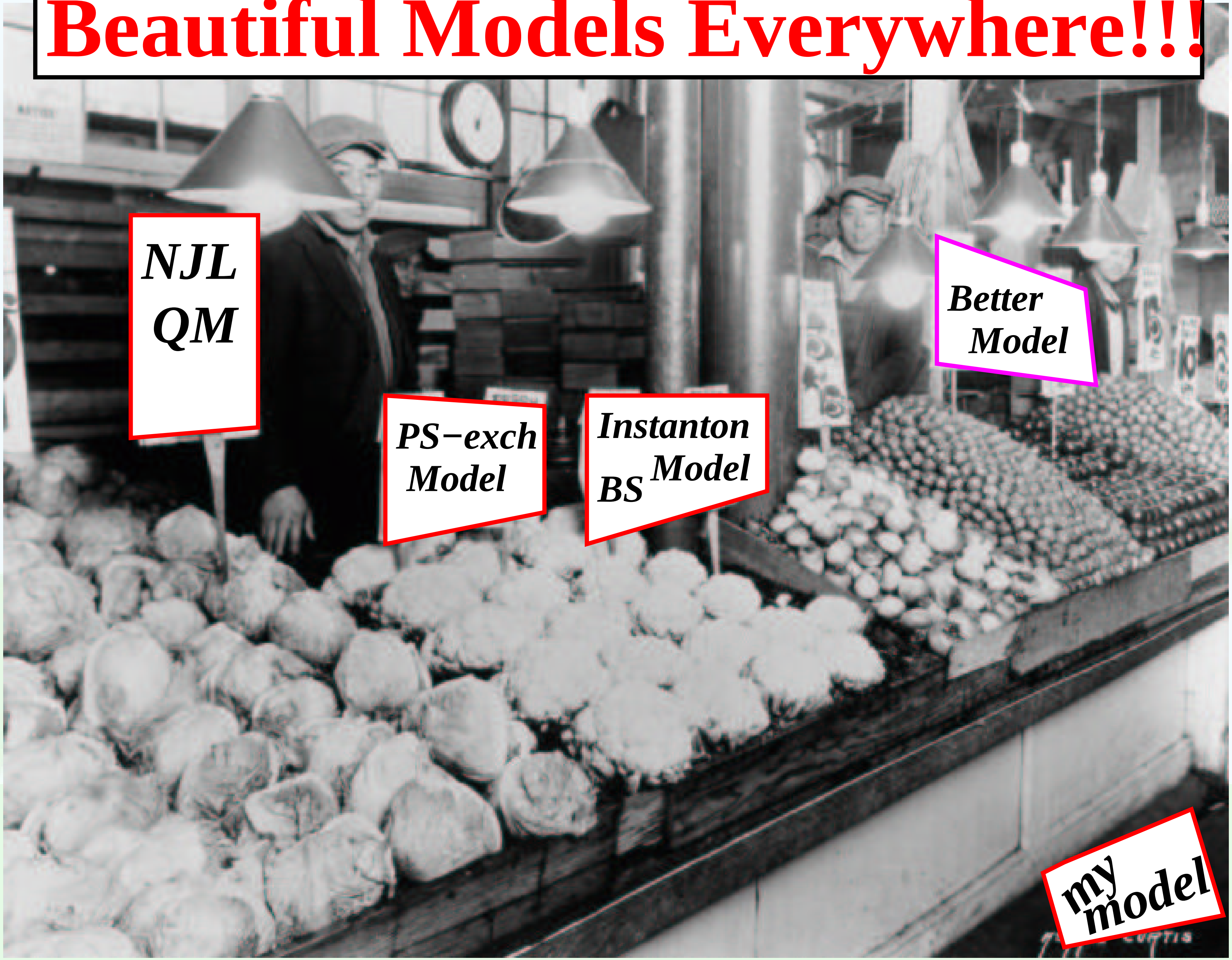
Beautiful Models Everywhere!!!



issues

(roughly)

Quark Models Comparison

# parameters	relativistic kinetic interaction	self−energy quark + meson loops	fixed spin structure S.S L.S	chiral symmetry π	results mesons baryons excited states	model
20	some kin.	meson	No	No	MBX	CQM (Capstick,...)
7	point form	partly	partly	No	MBX	PS exchange (Plessas,...)
4	yes (inst)	partly	partly	No	MBX	Instanton BSE (Metsch,....)
2	yes	quark	yes	π	MX	Coulomb BCS (Swanson,....)
2	yes	quark	sure(?)	No	MB(X)	Lattice (Morningstar,...)
4	yes	quark	yes	π	M(BX)	SDE (Roberts,....)

QCD in the Coulomb gauge

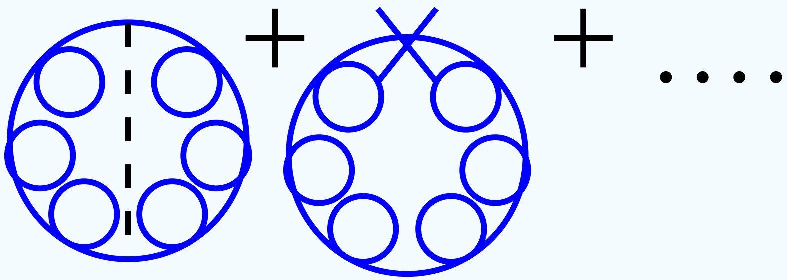
(Christ&Lee, Zwanziger, Swanson, Szczepaniak, Cotanch, ...)

- Separation of dynamics (massive gluons) and confining forces ("scalar" potential)
- Massive quarks through chiral symmetry breaking à la Bogoliubov-Valatin (minimizing the one-body energy).
- Perturbative expansion in massive quarks and gluons.

theory

The potential (Swanson and Szczepaniak)

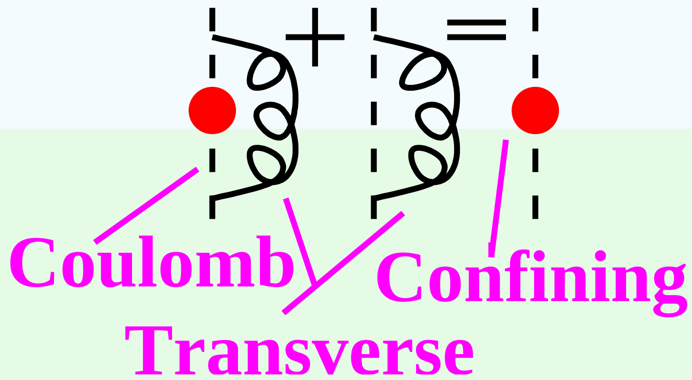
Correlated VACUUM:



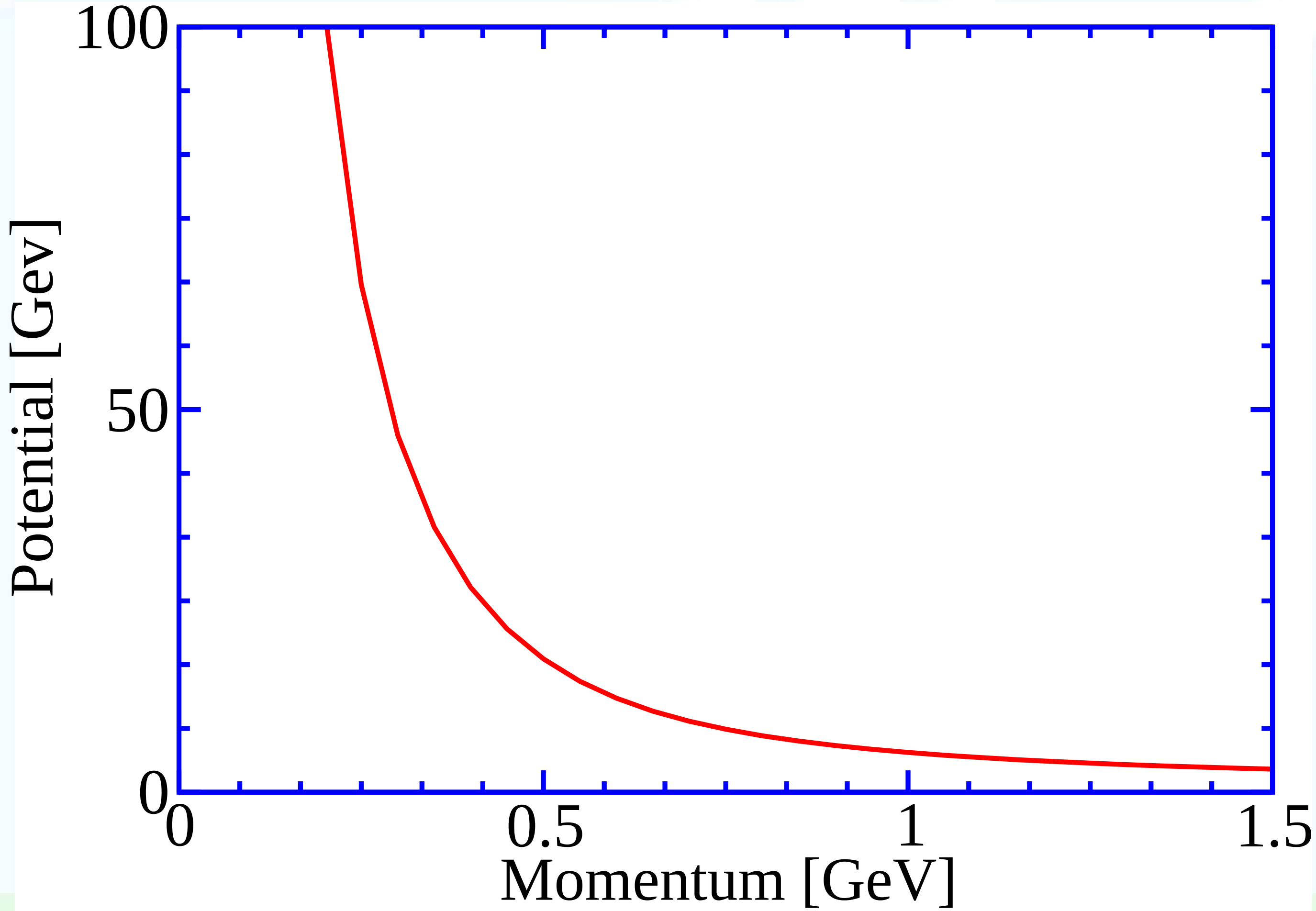
Vacuum energy

to be minimized

dressing the potential

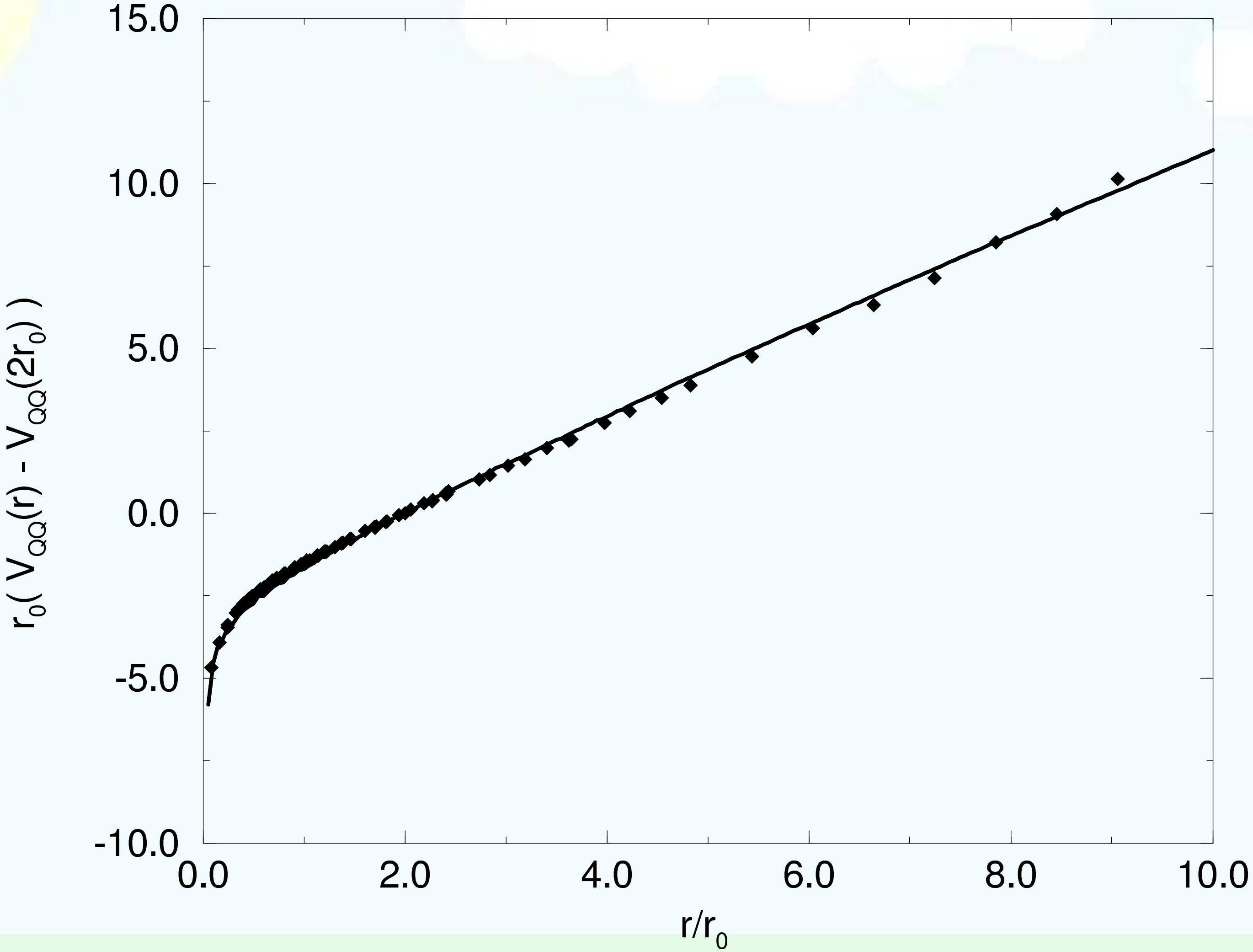


theory



The result: — $q^2V(q)$ **[GeV]**.

theory



The result in configuration space: — $V(R)$.

issues

Why non-relativistic approach fails

$$E = \frac{3\mu}{m_q} + 2m_q - \alpha\sqrt{2\pi\mu} + \sigma\sqrt{\frac{\pi}{2\mu}}$$

where μ is the wave function scale, e.g., $\exp\{-r^2/(2\mu)\}$.

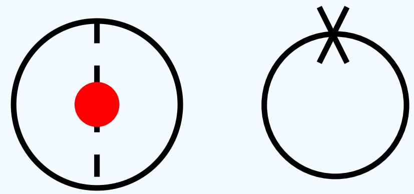
Bounded by either μ or m_q or $\sigma/\sqrt{\mu}$.

theory

Quark lines

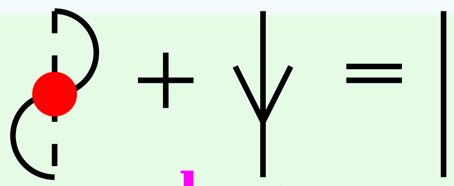
$$\sum_s u_s(m(p),p)\ u_s^\dagger(m(p),p)$$

Vacuum Energy:

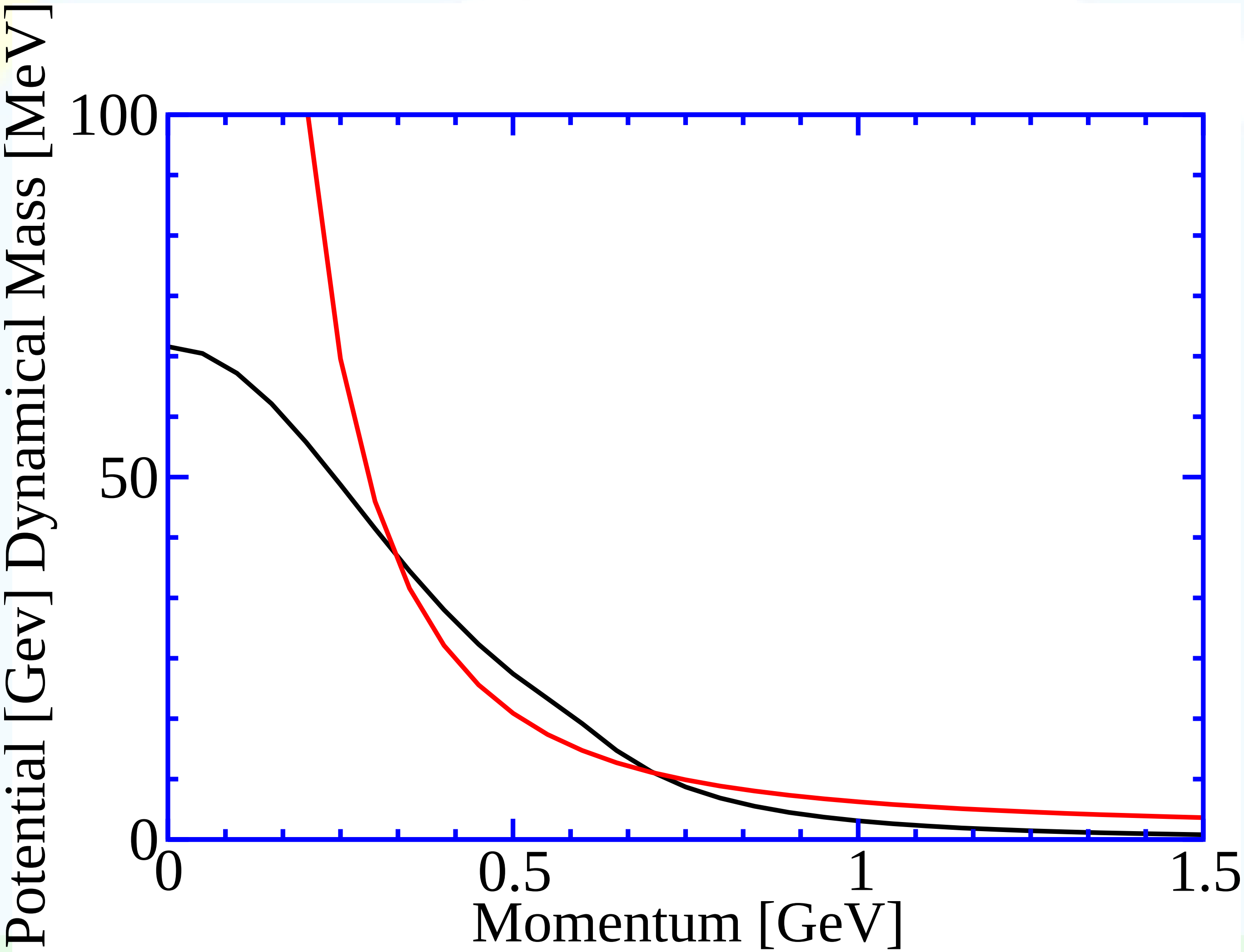


Gap equation

(minimizing the vacuum energy by varying m(



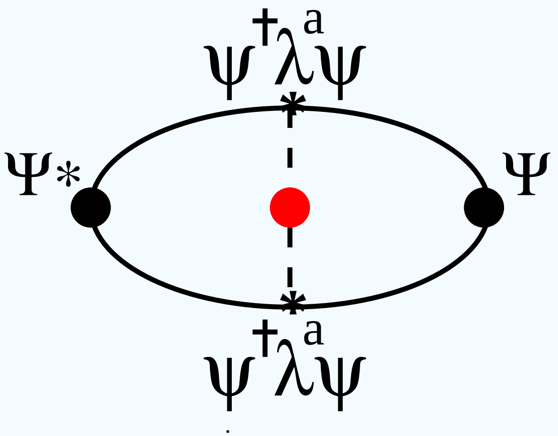
theory



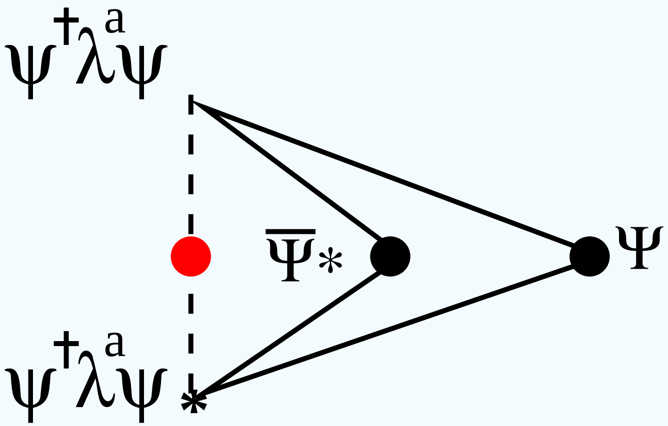
The results: — $q^2V(q)$ [GeV] and — $m(q)$ [MeV].

theory

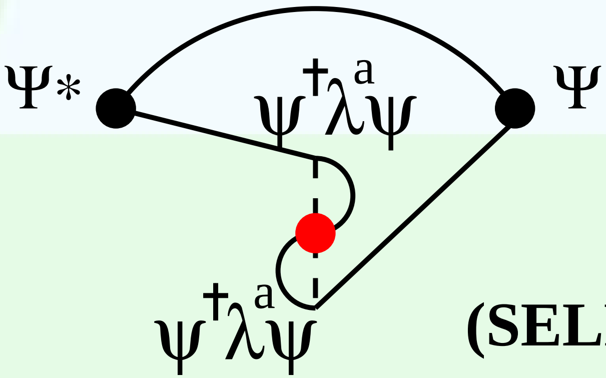
Mesons



(TDA)



(RPA(chiral symmetry))



(SELFENERGY (2x))

BEYOND (what else, what next)

- More complicated vacuum content: ggg, g^4, $q\bar{q}$, etc.
- Higher order vacuum diagrams. The transverse gluon?
- Short range interactions, spin-spin and renormalization (1S states)
- Coupling to decay channels: widths and shifts (data comparison?)
- Flavor mixing

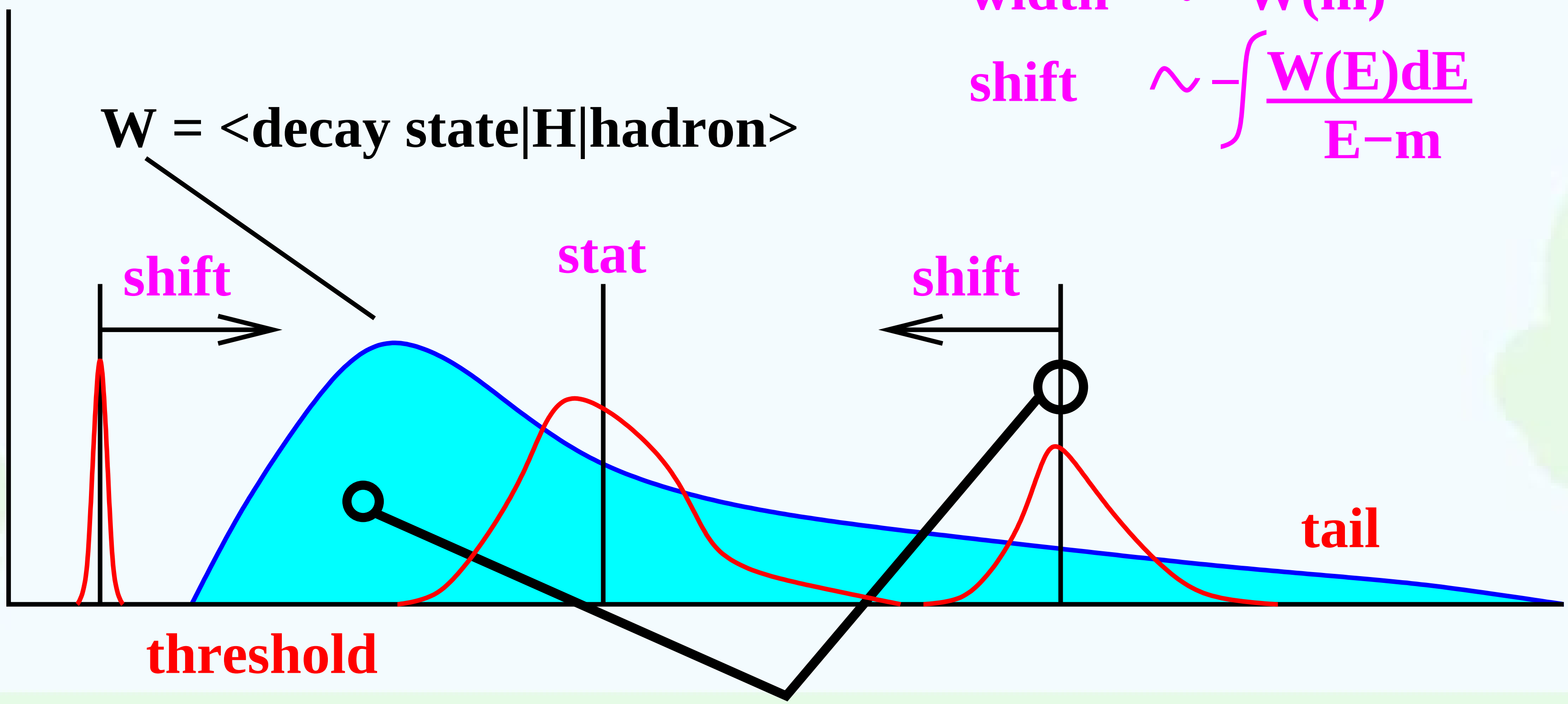
issues

WIDTHS and SHIFTS

(Comparing to data is not that straightforward)

$$\text{width} \sim W(m)$$

$$\text{shift} \sim -\int \frac{W(E)dE}{E-m}$$



coupling between the continuum and the hadron determines the width

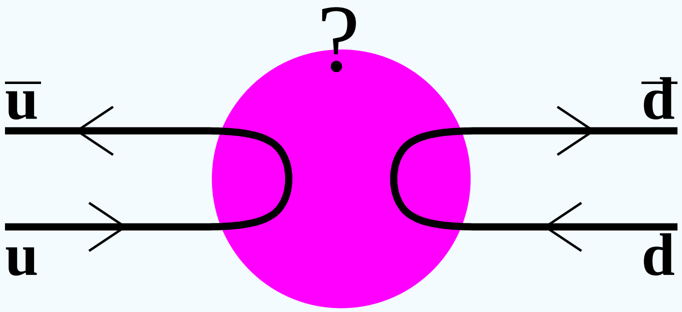
***width is proportional to the coupling to the decay state**
***shift depends on width, tail, threshold, and mass.**

issues:

Flavor Mixing (iso−scalars)

* How do they stay degenerate? $\boxed{\rho,\omega}$

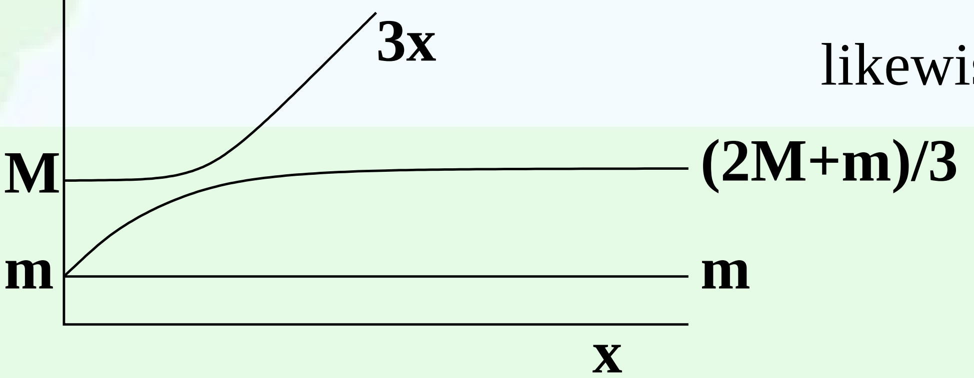
* How do they split? $\boxed{\eta,\eta' \quad \pi}$



$\bar{u},\bar{d},\bar{s}$

u,d,s

$$H_f = \begin{bmatrix} m+x & x & x \\ x & m+x & x \\ x & x & M+x \end{bmatrix}$$

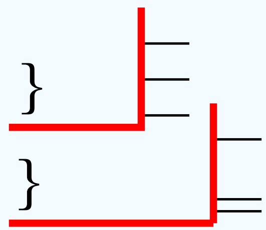


π^0 — π^+ — π^- — remain degenerate

$\{\pi, \eta, \eta'\}$

likewise:

$\{\rho, \omega, \phi\}$

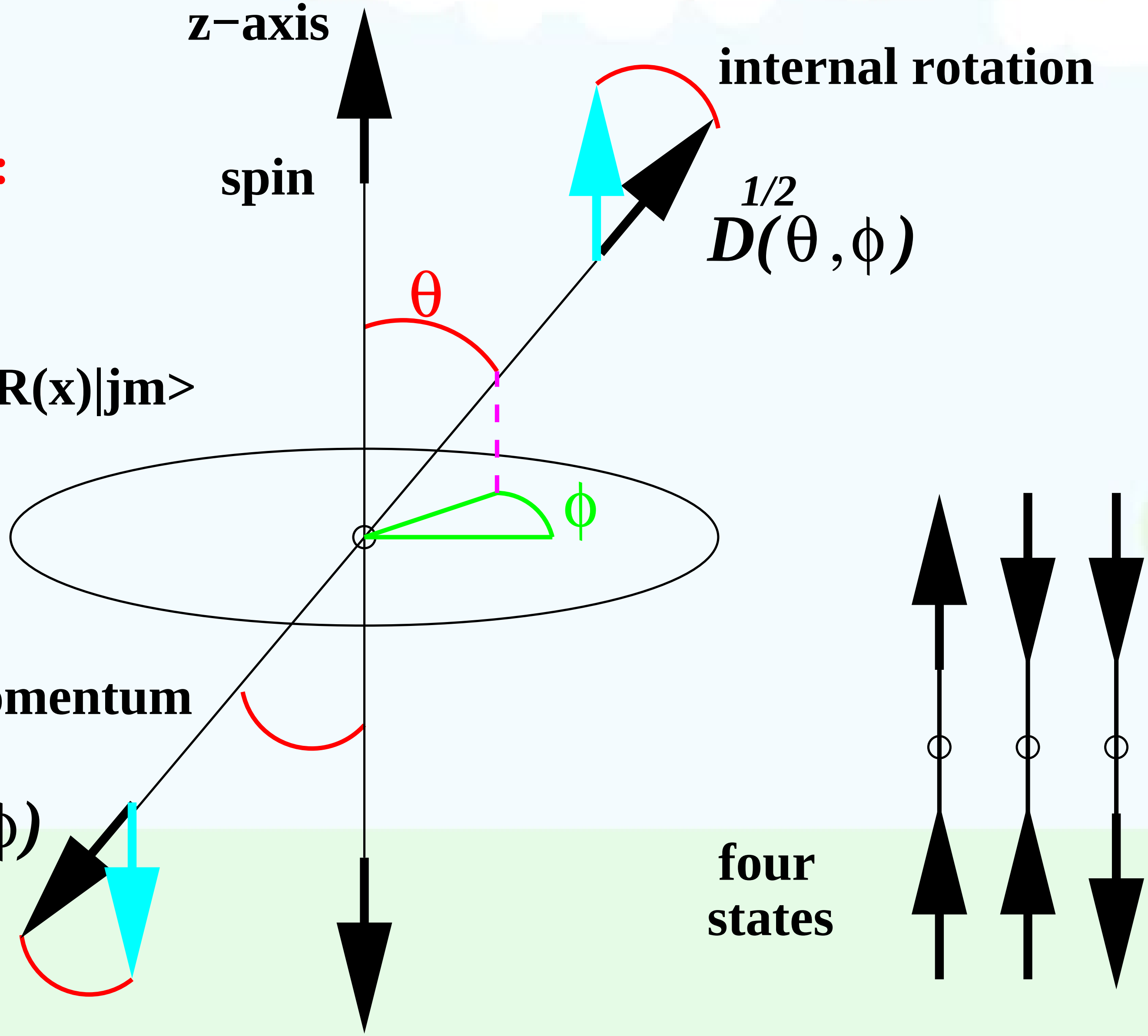


calculation

Helicity basis:

−invariant

$$D^{j}_{m'm}(x) = \langle jm'|R(x)|jm\rangle$$



calculation

$2 \otimes 2$ spin states, 4 cases ($J^{(PC)}$):

$$
\begin{aligned}
J^{(-[J],[J])} : (\text{Singlet}) &= \uparrow\downarrow + \downarrow\uparrow \\
J^{(-[J],-[J])} : (\text{Triplet}) &= \uparrow\uparrow \otimes Y_{lm} \\
J^{([J],[J])} : (\text{Triplet}^2) &= \uparrow\uparrow Y_{l'(m-1)} + \downarrow\downarrow Y_{l(m+1)}
\end{aligned}
$$

The last set corresponds the "S-D" mixing through tensor interaction.

calculation

PURE STATES

$J^{-[J][J]}$ [${}^1J_J$, $J \geq 0$]:

$$K_J(p,k) = V_J(1+\frac{m(p)}{E(p)}\frac{m(k)}{E(k)}) + \left(V_{J-1}\frac{J}{2J+1} + V_{J+1}\frac{J+1}{2J+1}\right)\frac{p}{E(p)}\frac{k}{E(k)}$$

0^{++}:

$$K(p,k) = V_0\frac{p}{E(p)}\frac{k}{E(k)} + V_1(1 + \frac{m(p)}{E(p)}\frac{m(k)}{E(k)})$$

$J^{-[J]-[J]}$ [${}^3J_J$, $J \geq 1$]:

$$K_J(p,k) = V_J(1+\frac{m(p)}{E(p)}\frac{m(k)}{E(k)}) + \left(V_{J-1}\frac{J+1}{2J+1} + V_{J+1}\frac{J}{2J+1}\right)\frac{p}{E(p)}\frac{k}{E(k)}$$

calculation

MIXED STATES

$J^{[J][J]}$ $[{}^3(J-1)_J, {}^3(J+1)_J\ ,\ J \geq 1]$:

$$
\begin{aligned}
K_{11}(p,k) &= V_J \frac{p}{E(p)} \frac{k}{E(k)} + \left(V_{J-1} \frac{J}{2J+1} + V_{J+1} \frac{J+1}{2J+1} \right) (1 + \frac{m(p)}{E(p)} \frac{m(k)}{E(k)}) \\
K_{22}(p,k) &= V_J \frac{p}{E(p)} \frac{k}{E(k)} + \left(V_{J-1} \frac{J+1}{2J+1} + V_{J+1} \frac{J}{2J+1} \right) (1 + \frac{m(p)}{E(p)} \frac{m(k)}{E(k)}) \\
K_{12}(p,k) &= \left(V_{J-1} - V_{J+1} \right) \frac{\sqrt{J(J+1)}}{2J+1} \left(\frac{m(p)}{E(p)} + \frac{m(k)}{E(k)} \right)
\end{aligned}
$$

calculation

Resolving the implicit spin structure:

$$u_s^\dagger(\mathbf{p}+\mathbf{q}/2)u_t(\mathbf{p}-\mathbf{q}/2) = \frac{(E'+m')(E+m)+p^2-q^2/4}{\sqrt{4EE'(E'+m')(E+m)}}\delta_{st} + \frac{\chi_s^* i(\vec{p}\times\vec{q})\cdot\vec{\sigma}\chi_t}{2\sqrt{4EE'(E'+m')(E+m)}}$$

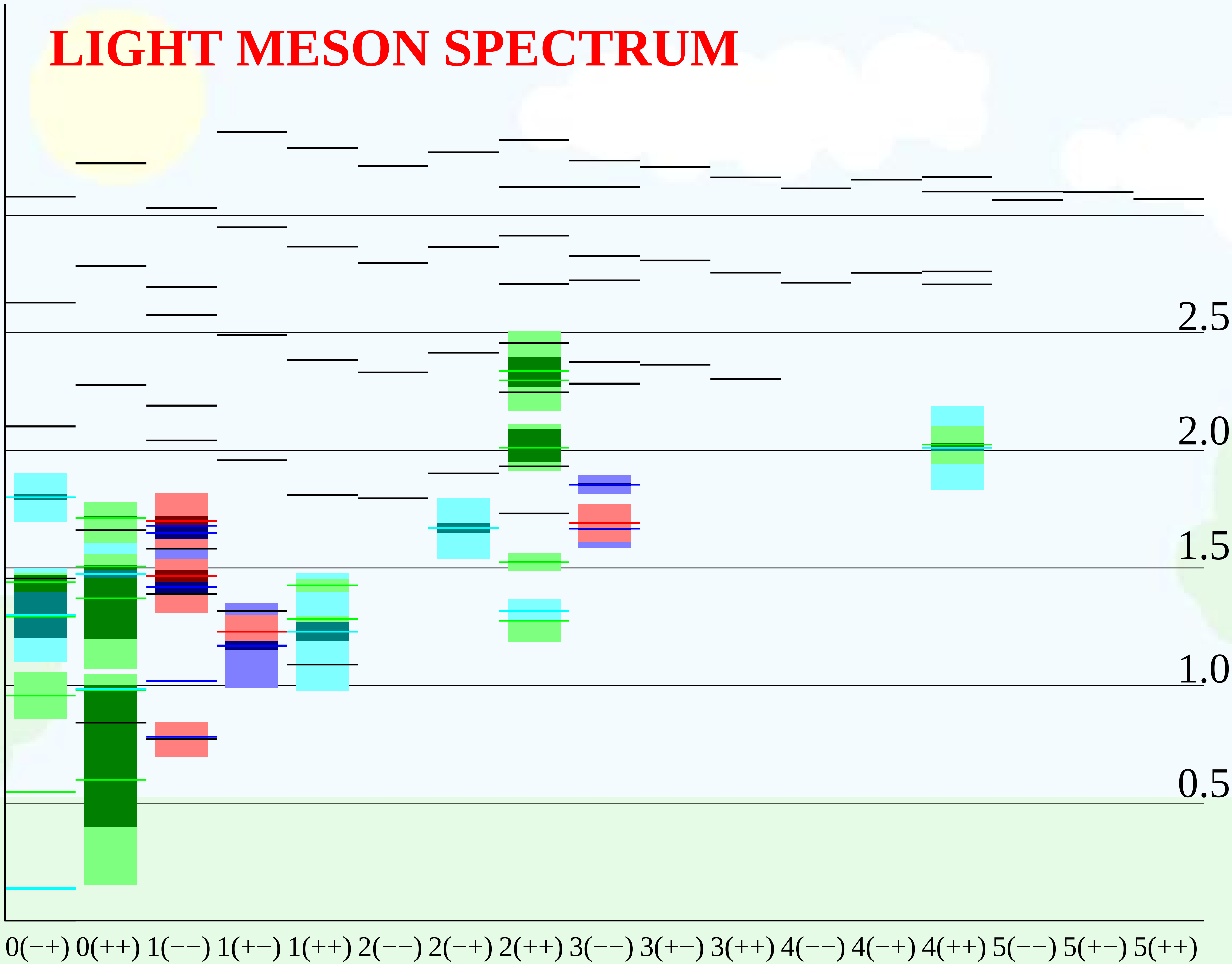
$q \ll p \ll E$:

$$V \sim |u_s^\dagger u_t|^2 \sim \delta_{st}\left(1-\frac{q^2}{8M^2}\right)+\frac{\chi_s^* i(\vec{p}\times\vec{q})\cdot\vec{\sigma}\chi_t}{4M^2}$$

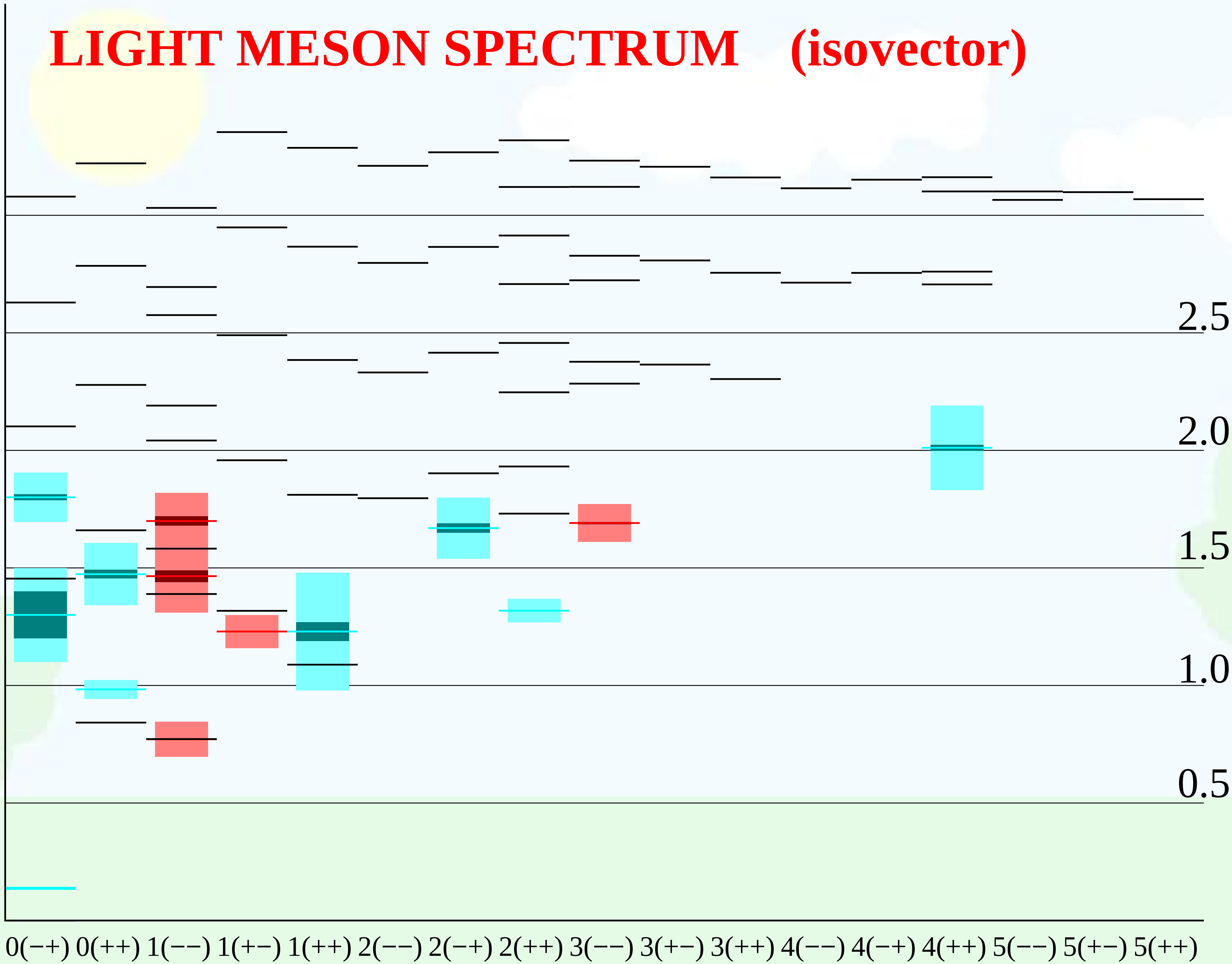
$p \ll q \ll E$ **(hard gluon):**

$$V \sim |u_s^\dagger u_t|^2 \sim \delta_{st}\left(1-\frac{5q^2}{8M^2}\right)+\frac{\chi_s^* i(\vec{p}\times\vec{q})\cdot\vec{\sigma}\chi_t}{4M^2}$$

LIGHT MESON SPECTRUM

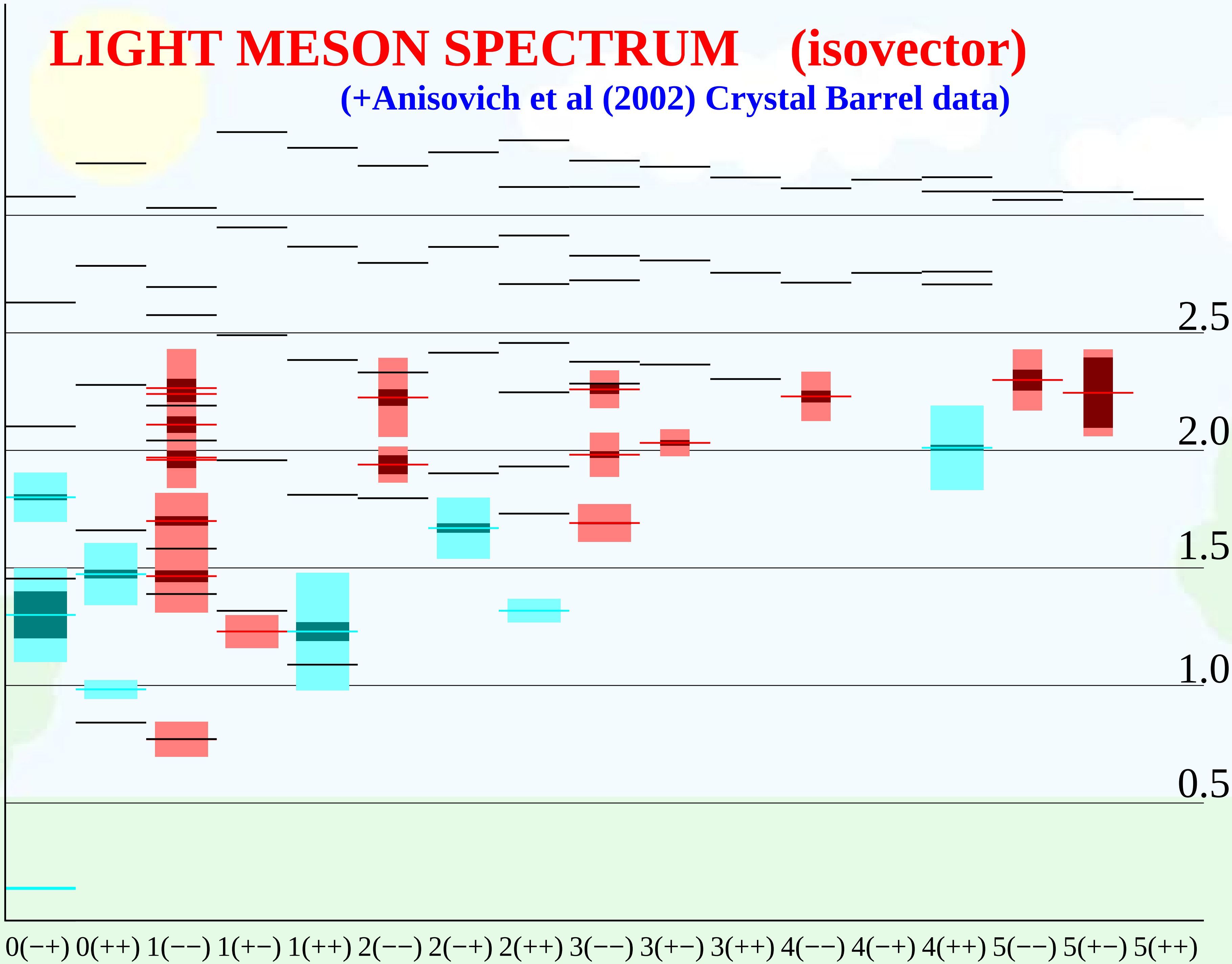


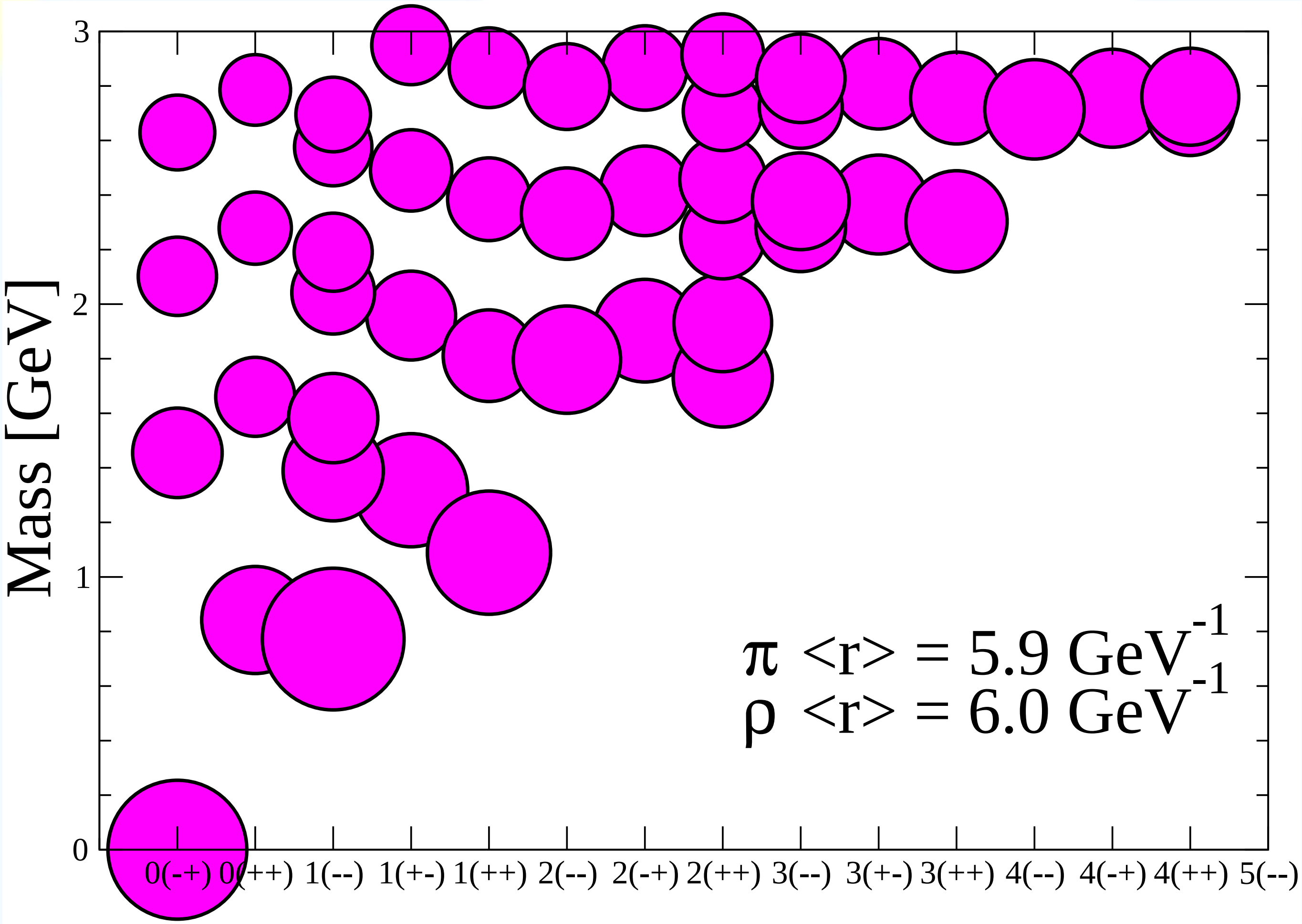
LIGHT MESON SPECTRUM (isovector)



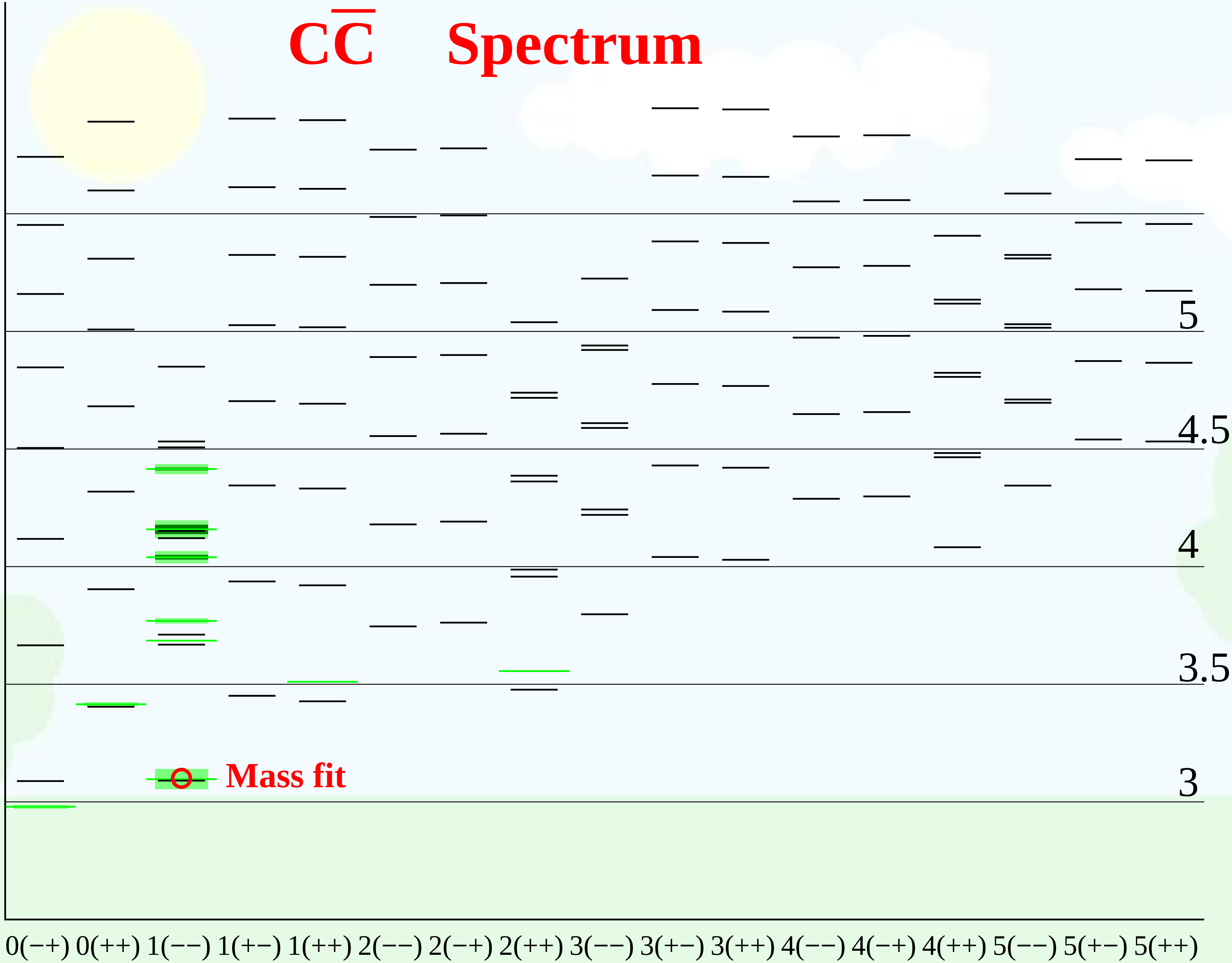
LIGHT MESON SPECTRUM (isovector)

(+Anisovich et al (2002) Crystal Barrel data)



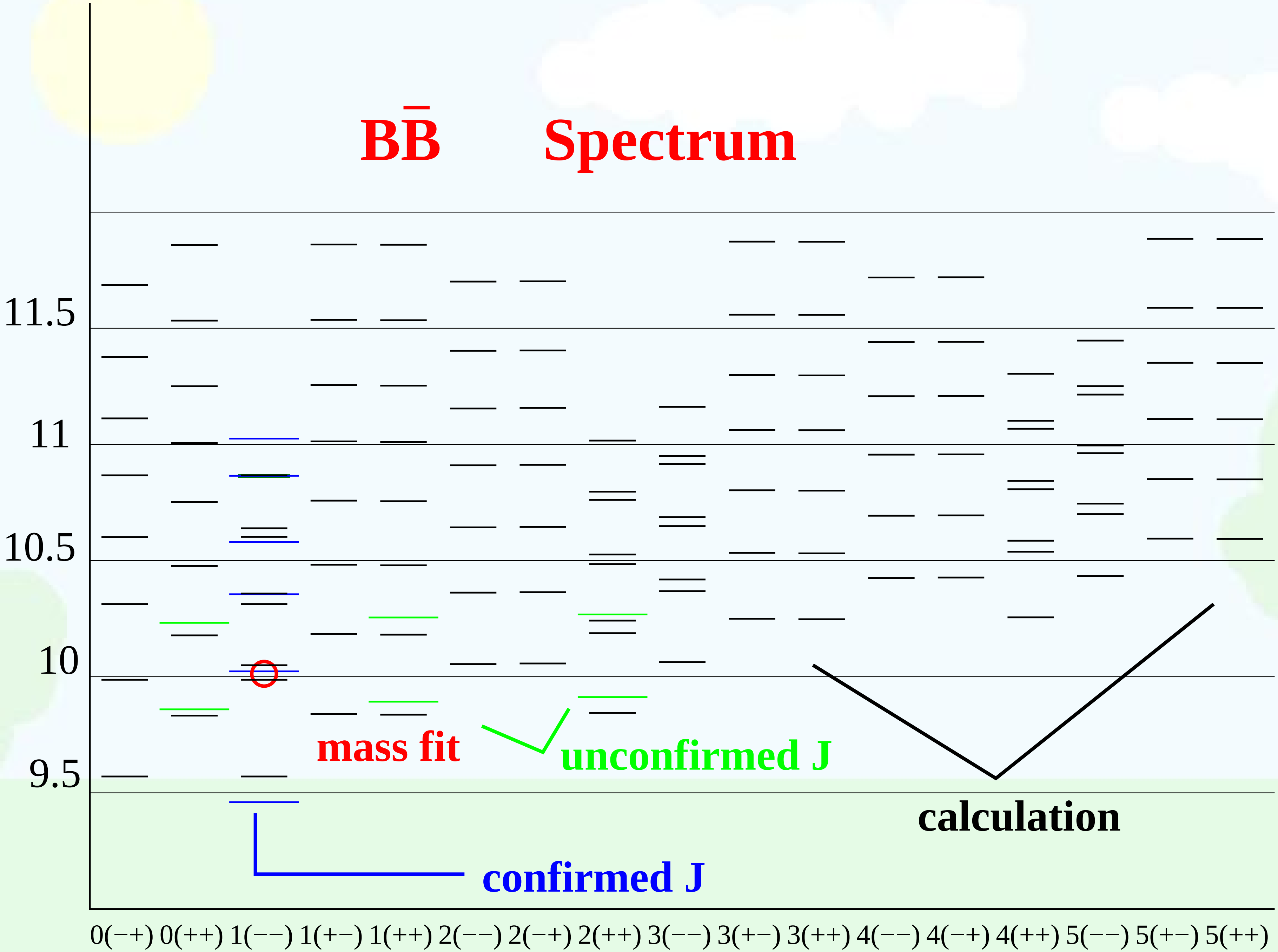


The radii $\sqrt{\langle r^2 \rangle}$ of the light mesons.
(Through Gaussian Fit Fourier Transform.)



CC̄ Spectrum
Mass fit
5
4.5
4
3.5
3
0(−+) 0(++) 1(−−) 1(+−) 1(++) 2(−−) 2(−+) 2(++) 3(−−) 3(+−) 3(++) 4(−−) 4(−+) 4(++) 5(−−) 5(+−) 5(++)

$B\bar{B}$ Spectrum



CONCLUSIONS

- Coulomb gauge: when a potential makes sense
- The chiral pion
- Full spectrum, singular approach, minimal parametric fitting
- Meson properties available
- Meson interaction and decay under investigation