Quantum phases and an anomaly of interacting fermionic atoms

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Two topics:

I. Breached pair superfluidity. [with F. Wilczek, P. Zoller, E. Gubankova, M. Forbes]

II. A quantum anomaly---chiral mass flow of atoms in p-wave resonance. [myself]
Breached pair superfluidity (BP)

Collaborators:  
M. Forbes (MIT graduate)  
E. Gubankova (MIT postdoc)  
F. Wilczek (MIT)  
P. Zoller (Innsbruck)

publications  
1. PRL 90, 047002 (2003)  
3. PRA 70, 033603 (2004)  

News story: “Odd particle out”,  
Phys. Rev. Focus (January 5, 2005; story 1)
Motivation: atomic Fermi gases

• BCS superfluidity of fermionic atoms
• BEC of molecules, BEC/BCS crossover, resonance models
• Pairing with mismatched fermi surfaces:
  ➢ Two spin components are separately conserved; different densities
  ➢ new pairing possibility? — “breached pairing”
  ➢ on-earth “atomic” simulator for color superconductivity in nuclear matter? (mismatched fermi surface in quark matter in neutron stars)
Different kinds of pairing

**BCS**
Bardeen-Cooper-Schrieffer (1957)

**LOFF**
Larkin and Ovchinnikov; independently Fulde and Ferrell (1964)

**Breached pairing**
Pairing occurs within the interior or exterior of a large Fermi ball. [This talk!].
Heuristic introduction to BP

*Recall BCS pairing*

momentum gap

\[ \kappa = \frac{\Delta}{v_F} \]

\[ \delta p_F \equiv p_F^\downarrow - p_F^\uparrow \]

when \( \delta p_F \gtrsim \kappa \), BCS impossible!
Breached Pair Superfluidity (BP)

BP state = a superfluid + a normal Fermi liquid at T=0; has gapped and gapless quasiparticle excitations.

[WVL, F. Wilczek, PRL (2003)]
Mean field theory of BP

Model:

\[ H = \sum_p \epsilon_{p\alpha} \psi_{p\alpha}^\dagger \psi_{p\alpha} + \sum_{pp'} V(p - p') \psi_{p\uparrow}^\dagger \psi_{-p\downarrow}^\dagger \psi_{-p'\downarrow} \psi_{p'\uparrow} \]

\[ \epsilon_{p\alpha} = \frac{p^2}{2m_{\alpha}} - \mu_{\alpha}, \quad \alpha = \uparrow, \downarrow \]

For real-space δ-like interaction

\[ V(q) = g = \frac{4\pi \hbar^2 a_s}{m} = \text{const} \]

\( (a_s < 0) \)
$BCS$

$BP$
Many body wavefunction

BCS vs BP

\[ |BCS⟩ = \prod_p (u_p + v_p \psi_p^{\uparrow} \psi_p^{\downarrow}) |0⟩ \]

\[ |BP⟩ = \prod_{p \not{\in} \text{breach}} (u_p + v_p \psi_p^{\uparrow} \psi_p^{\downarrow}) \prod_{p \in \text{breach}} \psi_p^{\uparrow} |0⟩ \]

where

\[
\begin{pmatrix}
u_p^2 \\
v_p^2
\end{pmatrix} = \frac{1}{2} \left( 1 \pm \frac{\epsilon_p^+}{\sqrt{\epsilon_p^{++} + \Delta_p^2}} \right)
\]

“breach” region: \( p_\Delta^- \leq |p| \leq p_\Delta^+ \)
How stable?

The stability of BP criticized by:


Both are correct, but are done for a short-range delta-interaction.

Stability issue overcome and clarified in:

our latest [PRL 94, 017001 (2005)]

Need
1. a finite or long range interaction; or
2. a momentum cutoff

$R^* \gtrsim k_F^{-1}$

effective range
inter-atom distance
Effective range in real atomic gases

From D. Petrov, talk given at KITP Conference: Quantum gases 2004:

<table>
<thead>
<tr>
<th></th>
<th>$R_o$ [Å]</th>
<th>$B_0$ [G]</th>
<th>$\Delta_B$ [G]</th>
<th>$\frac{\partial E_{res}}{\partial B}$</th>
<th>$a_{bg}$ [Å]</th>
<th>$R^*$ [Å]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6$Li</td>
<td>30</td>
<td>543.25</td>
<td>0.1</td>
<td>$2\mu_B$</td>
<td>32</td>
<td>19000</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>45</td>
<td>907</td>
<td>1</td>
<td>$3.7\mu_B$</td>
<td>33</td>
<td>260</td>
</tr>
<tr>
<td>$^{87}$Rb</td>
<td>85</td>
<td>1007.4</td>
<td>0.17</td>
<td>$2.5\mu_B$</td>
<td>60</td>
<td>320</td>
</tr>
<tr>
<td>$^{133}$Cs</td>
<td>100</td>
<td>19.8</td>
<td>0.005</td>
<td>$0.55\mu_B$</td>
<td>160</td>
<td>13000</td>
</tr>
</tbody>
</table>

[http://online.itp.ucsb.edu/online/gases_c04/petrov/]

Gas density: $n \sim 10^{14}\text{cm}^{-3} \Rightarrow k_F^{-1} \sim 1.0\mu\text{m}$
Case of strong coupling, short-range interaction, and equal mass

Quantum Monte Carlo results found:

A homogeneous, spin-polarized gapless superfluid [that is a BP] is favored against phase separation in real space.

[J. Carlson, Sanjay Reddy, cond-mat/0503256]
How to realize in atomic gases

A. Hetero-nuclear mixture of two species

$^6\text{Li} + ^{40}\text{K}$, $^6\text{Li} + ^{86}\text{Rb}$, ...

**Make two species of unequal densities!**

Hetero-nuclear resonance to generate attractive interactions.

B. Lattice atomic gases
Proposed experiment of fermionic atoms on lattice

[WVL, F. Wilczek, and P. Zoller, PRA (2004)]

Incoherent & different densities or coherent by Rabbi oscillation but detuned

Mismatched fermi surfaces

Hopping matrix elements:

\[ t_\uparrow \gg t_\downarrow , \quad t_\alpha \propto \frac{1}{m_\alpha} \]
Key features of Breached Pair

- coexisting superfluid and normal components at T=0;
- phase separated in momentum space;
- both gapped and gapless quasiparticle excitations.

relevances to reality

- realizable with cold atoms;
- may occur as a color superconductor in quark matter such as neutron stars
- “… other scenarios for uncondensed electrons should be considered, such as ‘interior gap [BP] superfluidity’” for the heavy-fermion superconductor CeCoIn5 [quote M. A. Tanatar, Louis Taillefer, et al. cond-mat/0503342]
Signature of breached pair superfluidity

(A quantum phase transition from BCS to BP)
Part 2. Chiral anomaly of an atomic Fermi gas
in p-wave resonance

[WVL, submitted for publication,
to be posted cond-mat/0503???]
p-wave Feshbach experiments

$^{40}$K atoms in $|\frac{9}{2}, -\frac{7}{2}\rangle$. $T_F \sim 1 \mu K \sim 0.01 G \times \mu_B$.

[Regal, Ticknor, Bohn and Jin, PRL (2003)]

$^{6}$Li in $|\frac{1}{2}, \frac{1}{2}\rangle + |\frac{1}{2}, -\frac{1}{2}\rangle$. $T_F \sim 10 \mu K \sim 0.1 G \times \mu_B$

[J.Zhang, C. Salomon, et al., cond-mat/0406085]
Anisotropy in p-wave Feshbach resonances

p-wave: $l = 1, m = -1, 0, +1$

Calculated

Experimentally observed

[by C. Ticknor, et al., PRA (2004)]

Fermi temperature: $T_F \sim 1 \mu K \sim 0.01 G \cdot \mu_B$
Theoretical modeling

strongly anisotropic interactions;
separate x,y orbitals from z orbital
Planar \( p \)-wave model for fermionic atoms

**Focus:**
On \( p_x \) and \( p_y \) orbital interactions;
All fermions in a single spin state.

**Model:**

\[
H = \sum_\mathbf{k} \varepsilon_\mathbf{k} a_\mathbf{k}^\dagger a_\mathbf{k} + \frac{g}{2\mathcal{V}} \sum_{\mathbf{qkk'}} \mathbf{k} \cdot \mathbf{k'} a_{\mathbf{q}+\mathbf{k}}^\dagger a_{\mathbf{q}-\mathbf{k'}}^\dagger \mathbf{a}_{\frac{\mathbf{q}}{2}+\mathbf{k}} - \mathbf{k'} \mathbf{a}_{\frac{\mathbf{q}}{2}-\mathbf{k}} + \mathbf{k} \mathbf{a}_{\frac{\mathbf{q}}{2}+\mathbf{k'}} - \mathbf{k'} \mathbf{a}_{\frac{\mathbf{q}}{2}-\mathbf{k'}}
\]

**Notation:**

★ boldface vector: \( \mathbf{k} = (k_x, k_y, k_z) \)
★ arrow vector: \( \mathbf{k} = (k_x, k_y) \) = planar vector
★ \( \varepsilon_\mathbf{k} = \frac{k^2}{2m} - \mu \)
★ \( \mathcal{V} \) = space vol.; \( g \) = coupling
The order parameter

p-wave pair operator:

\[ \vec{\Phi}_q = -\frac{g}{\sqrt{V}} \sum_k \vec{k} a_{\frac{q}{2}-k} a_{\frac{q}{2}+k}. \]

Complex, 2-component vector \( \Rightarrow \) 4 real variables

Parameterization:

\[ \langle \vec{\Phi} \rangle \equiv \begin{pmatrix} \langle \Phi^x_{q=0} \rangle \\
\langle \Phi^y_{q=0} \rangle \end{pmatrix} = \rho e^{i\vartheta} \begin{pmatrix} 1 \\
e^{-i\varphi} \end{pmatrix} \begin{pmatrix} \cos \chi \\
i \sin \chi \end{pmatrix}, \]

\[ 2 \times 2 \]

Pauli matrix

\( \vartheta = \) overall phase. \( \varphi = \) rotation angle in orbital space

\( \vartheta, \varphi \) are Goldstone bosons---gapless collective excitations.
Calculated effective potential (free energy at $T=0$)

\[
\langle \Phi \rangle = \rho e^{i\vartheta} e^{i\sigma_2 \varphi} \begin{pmatrix} \cos \chi \\ i \sin \chi \end{pmatrix}
\]

(set $\vartheta = \varphi = 0$)

Units of:
- energy $= \epsilon_F = \mu$
- length $= k_F^{-1}$

Energy minima at:
\[
\rho \neq 0
\]
\[
\chi = \pm \frac{\pi}{4}, \, \pi \pm \frac{\pi}{4}
\]

Order space coordination:
\[
(\phi^x, \, i\phi^y) = (\rho \cos \chi, \, -\rho \sin \chi)
\]
Analytical expression of the effective potential

\[ V[\rho, \chi] = \frac{\rho^2}{2|g|} - \int \frac{d^3 k}{2(2\pi)^3} \left[ \sqrt{\epsilon_k^2 + \rho^2 (k_x^2 \cos^2 \chi + k_y^2 \sin^2 \chi)} - |\epsilon_k| \right] \]
Axial superfluid

\[ \langle \tilde{\Phi} \rangle = \frac{\rho_0}{\sqrt{2}} \left(1 \pm i\right) \quad \text{for} \quad \chi = \pm \frac{\pi}{4} \]

Known as:

\( p_x + ip_y \) state; axial state; Anderson-Brinkman-Morel state (He-3 A phase);

In other p-wave models:

first predicted by Ho and Diener, cond-mat/0408468 and independently by Gurarie, Radzihovsky, Andreev, cond-mat/0410620

Orbital angular momentum (macroscopic):

\[ L_z^{\text{total}} = \pm \frac{N_0}{2} \hbar \quad \text{for} \quad \chi = \pm \frac{\pi}{4}. \]

\[ N_0 = \text{of atoms in condensed pairs} \]
Effective field theory of the order parameter field

\[ \langle \tilde{\Phi}(\mathbf{r}) \rangle = \left( \frac{\langle \Phi^x(\mathbf{r}) \rangle}{\langle \Phi^y(\mathbf{r}) \rangle} \right) = \rho_0 \left( \begin{array}{c} \cos \chi(\mathbf{r}) \\ i \sin \chi(\mathbf{r}) \end{array} \right) \]  

(spacially nonuniform)  

(\rho_0 = \text{constant})

Free energy functional

\[ F[\chi] = \frac{\rho_0^2}{C_\phi} \int d^3 \mathbf{r} \left[ \frac{1}{2} (\nabla \chi)^2 + \frac{1}{4\xi^2} \left( 1 + \cos(4\chi) \right) \right] \]  

[sine-Gordon theory in 3D Euclidean space]  

minimized by \( \chi_\pm = \pm \frac{\pi}{4} \)

Physical understandings:

1. \( \rho_0 = \Delta_0/k_F; \) \( \Delta_0 = \text{maximum energy gap} \)
2. \( C_\phi \approx 1/(mk_F), \) \( \text{nonuniversal (renormalizable),} \)
   related to the mass of p-wave pairs (molecules).
3. \( \xi \approx \Delta_0/v_F = \text{coherence length} \)
Domain walls as topological defects ---solitons

Solution to field equation

\[ \chi_{\pm}(r) = \pm \arctan \left( \tanh \left( \frac{x}{\xi} \right) \right) \]

\[ \chi = + \frac{\pi}{4} \]

\[ \frac{L_z}{\rho^2} = \sin 2\chi \]

use

\[ \vec{L} = -i \langle \Phi \rangle^* \times \langle \Phi \rangle \]

domain wall energy per unit area

\[ = \frac{\Delta_0^2}{(2C_\phi \xi k_F^2)} \sim \frac{\Delta_0}{\xi^2} \]

APS March 2005 27
Quantum anomaly
[chiral mass flow of atoms in the groundstate]

Heuristic steps to discover the anomaly
Anomaly (II)

View of three macro-boxes

A soliton slowly varies in space!

\[ \Delta_y \text{ or } \Phi^y \quad (\text{both } \propto \sin \chi) \]

\[ O(\xi) \]

left box \( p_x - ip_y \)

domain wall box \( p_x \text{ state} \)

right box \( p_x + ip_y \)

Important lengths \( \xi \gg k_F^{-1} \), \( (k_F^{-1} \sim 1/\sqrt{\text{density}}) \)
**Anomaly (III)**

The energy gap of quasiparticles (fermionic, atomic states)

$p_x + i p_y$ state  \hspace{1cm} $p_x$ state

For both states, gap vanishes at the north & south poles on z-axis!

**spectrum:**

$$E_k = \sqrt{\left( \frac{k^2}{2m} - \mu \right)^2 + \rho_0^2 (k_x^2 \cos^2 \chi + k_y^2 \sin^2 \chi)}$$

Representative point: $\mathbf{k} = (0, k_F, 0)$
Anomaly (IV): Gapless fermion bound states

Solve Bogoliubov-de Gennes equations (similar to Jakiew-Rebbi problem of Dirac fermions)

\[ E = \pm \epsilon_{\mathbf{k}||} \quad \text{for } k_y \geq 0 \text{ or } \leq 0 \]

with \( \mathbf{k}_{||} = (0, k_y, k_z) \)

\[ \text{found bound states analytically for } \xi \gg k_F^{-1} \]
Anomalous quantum mass flow of atoms in the groundstate *(No additional external force!)*

mass current per unit z-length:

\[ \mathbf{j} = -\frac{\hbar k_F^3}{6\pi^2} \hat{e}_y, \]

*(\hbar restored for clarity)*
Summary

Breached Pair Superfluidity

[courtesy of Phys. Rev. Focus (Jan 2005)]

Domain wall quasiparticles and chiral anomaly in p-wave

[submitted for publication]

APS March 2005