The Nature of Stars

• **Goals:**
  - Measuring distances to stars using parallax and luminosity.
  - The “magnitude” scale.
  - The Hertzsprung-Russell Diagram.

• **Stellar Distances**
  - **Stellar Parallax**
    
    **Figure 19-2**
    
    • Nearby objects “move” relative to the background if we change our position.
    
    • Distance of a nearby star can be measured through parallax (the orbit of the Earth changes our position).
    
    • When the parallax angle is 1 arcsecond the distance of a star is defined to be 1 parsec (3.09x10^{13} km, 3.26 ly).

    \[
    d = \frac{1}{p}
    \]

    - d: distance (in parsecs)
    - p: parallax angle (arcsecs)
Parallax of Proxima Centauri

\[ p = 0.772 \quad \text{(parallax angle)} \]

\[ d(\text{parsec}) = \frac{1}{p \text{ (arcsec)}} \]

\[ = \frac{1}{0.772} = 1.30 \text{ pc (4.2 ly)} \]

What is the furthest star for which we can measure its parallax with HST?

- Resolution of HST telescope is 0.05 arcsec
  \[ d = \frac{1}{p} \]
  \[ = \frac{1}{0.05} = 20 \text{ pc} \]
- Parallax can be measured only for a small number of nearby stars (we are 8 kpc from the center of our Galaxy).
- We can identify about 200 parallax angles from the Earth (need satellite missions such as Hipparcus).
• **Stellar Motions**

  – Gravity causes stars to move

  **Box 19-1**

  • Stars can move in any direction - we can measure **tangential** and **radial** components.
  • Tangential (perpendicular to line of sight): \( v_t \).
  • Radial (along our line of sight): \( v_r \).
  • We measure the angle a star moves as a function of time (**proper motion**).

  \[
  v_t = 4.74 \mu d
  \]

  • \( v_t \): tangential velocity (km s\(^{-1}\))
  \( \mu \): proper motion (arcsecs per year)
  \( d \): distance (parsecs)

  • We measure proper motion by observing the motion of the star relative to the background over a number of years.

  • We measure radial velocities through the doppler shift (redshift of spectral lines).

  \[
  \frac{\lambda - \lambda_o}{\lambda_o} = \frac{v_r}{c}
  \]

  • \( \lambda \): wavelength observed
  \( \lambda_o \): wavelength if star was not moving
  \( v_r \): radial velocity of the star
Example: How far does a star move (in angle) per year?

- A typical star moves with a velocity of 10 km s\(^{-1}\) (31,000,000 km in a year).
- If this star was Proxima Centauri (4.2 light years away) then

\[
v_t = 4.74 \mu \text{ d}
\]

\[
\mu (\text{arcsec per year}) = \frac{v_t (\text{km s}^{-1})}{4.74 \text{ d (parsecs)}}
\]

\[
= \frac{10}{4.74 \times 4.2 \times 0.306}
\]

\[
= 1.6 \text{ arcsec per year}
\]

- Typical motions of stars are about 0.1 arcsecs per year.
• **Luminosity vs Distance**
  
  – **Apparent luminosity**
  
  **Figure 19-4**
  
  • The intrinsic luminosity \( (L) \) of a star depends on its mass, temperature, composition.

  • The observed luminosity (apparent brightness) depends on a star's distance.

  \[
  b = \frac{L}{4\pi d^2}
  \]

  \( b \): apparent brightness (W m\(^{-2}\))

  \( L \): luminosity (W)

  \( d \): distance (m)

  • Simply the “inverse square law” (e.g. the relation between flux and luminosity).

  • **Increase** distance \( \rightarrow **decrease** \) brightness.

  – **Luminosity relative to the Sun**

  • Easier to relate the luminosity of a star to that of the Sun.

  \[
  L_O = 4\pi d_O^2 b_O
  \]

  • \( L_\odot \): Sun’s luminosity

  • \( d_\odot \): Sun’s distance (1 AU)

  • \( b_\odot \): Sun’s apparent brightness
– Luminosity Relative to the Sun

\[ \frac{L}{L_o} = \left( \frac{d}{d_o} \right)^2 \frac{b}{b_o} \]

- To determine a star’s luminosity relative to the Sun we need the distance of the star relative to the Sun-Earth distance and the apparent luminosity relative to the Sun.

- Example:

  \[ \frac{d_1}{d_2} = \sqrt{\frac{L_1}{L_2}} \]

  \[ \sqrt{\frac{b_1}{b_2}} \]

  G2 star (like the Sun) appears $10^3 \times$ fainter

  \[ \frac{d_1}{d_2} = \sqrt{\frac{1}{1/10^{-4}}} \]

  \[ d_2 = 10^2 \ d_1 \]

  Second star must be 100x further away than the Sun from the Earth ($d_2=100$ AU).

- Most stars are less luminous than the Sun.
• **Stellar Luminosity Function**
  
  – **Distribution in Star Luminosities**

  **Figure 19-5**

  • Range of stellar luminosities - from brown dwarfs (failed stars) to very massive stars \((10^6 \times \text{the Sun’s luminosity})\).

  • We can describe the distribution of stellar luminosities using the luminosity function.

  • This gives the number density of stars (number per cubic parsec) as a function of luminosity.
• **Measuring a Star’s Brightness**
  
  – **The magnitude scale**

  **Figure 19-6**

  • The range of stellar brightness (and galaxy) is very large ($L_\odot = 3.9 \times 10^{26}$ W).

  • We often use magnitudes (logarithmic scale) to describe the brightness of an object.

  \[ m = -2.5 \log_{10} (b) \]

  • The logarithmic scale is the same way the sensitivity of the eye works.

  • Stars half as bright are about 1 magnitude fainter.

  \[ m_1 - m_2 = -2.5 \log_{10} (b_1) + 2.5 \log_{10} (b_2) \]

  \[ = -2.5 \log_{10} \left( \frac{b_1}{b_2} \right) \]

  if $b_2 = 0.5 \times b_1$ then

  \[ m_1 - m_2 = -2.5 \log_{10} \left( \frac{b_1}{0.5 \times b_1} \right) \]

  \[ = -0.75 \]

  • Bright object → more **negative** magnitude.

  • 100x brighter → -5 in magnitude.
- **Apparent magnitude (m)**
  - Used to specify the apparent or perceived brightness of a star. The apparent magnitude depends both on the star’s intrinsic brightness (absolute magnitude) and the star’s distance.

- **Examples:**
  - Vega (α Lyrae) has $m_v = 0$
  - Sun: $m_v = -26.8$
  - Moon: $m_v = -12.6$
  - Venus at maximum brightness: $m_v = -4.4$
  - Sirius (the brightest star): $m_v = -1.4$
  - The faintest stars we can see with the unaided eye: $m_v = 6$.

- **The apparent magnitude (m) and brightness (b) of 2 stars are related by the formula:**
  - $m_2 - m_1 = 2.5 \log \left( \frac{b_1}{b_2} \right)$
– **Apparent vs Absolute Magnitude**

- When we compare the brightness of two stars we want to do it with them at the same distance.
- The absolute magnitude of a star is defined to be the same as the apparent magnitude of the star at 10 parsec.
- Absolute Magnitude is a capitalized M.
- Example:
  - Sun: $M_v = 4.8$ ($m_v = -26.8$)
  - Sirius: 1.4

– **The absolute and apparent magnitude is related by**

$$m - M = 5 \log_{10}(d) - 5$$

- $d$: distance (parsecs)
- $m$: apparent magnitude
- $M$: absolute magnitude
- $m-M$ is called the distance modulus (it tells the distance of an object).
Example: Using magnitudes

- A pair of stars 100pc away form a binary system. Star A has an apparent magnitude of $m_A = 8$ and star B $m_B = 5$. What is their total magnitude and absolute magnitude?
  
  \[ m_1 = -2.5 \log_{10} (b_1) \]

- Calculate the fluxes and add them
  
  \[ b_1 = 10^{-2.5} \]
  \[ b_2 = 10^{-2.5} \]
  
  \[ b_1 = 6.3 \times 10^{-4} \]
  \[ b_2 = 6.3 \times 10^4 \]
  
  \[ = 0.01 \]

- Recalculate magnitude
  
  \[ m_{\text{Total}} = -2.5 \log_{10} (b_1 + b_2) \]
  
  \[ = -2.5 \log_{10} (6.3 \times 10^{-4} + 0.01) \]
  
  \[ = -2.5 \log_{10} (0.01063) \]
  
  \[ = 4.93 \]

- Calculate distance modulus
  
  \[ m - M = 5 \log_{10} (d) - 5 \]
  
  \[ M = 4.93 - 5 \log_{10} (100) - 5 \]
  
  \[ M = -10.07 \]
– Measuring Colors of Stars

**Figure 19-7, 19-8**

- The spectrum of a star is dependent on its surface temperature.
- Cool stars emit most of their light at long wavelengths and hot stars at short wavelengths (Wein’s law).
- Measuring the flux from a star at different wavelengths we can measure its color.

– Photometry

- Technique astronomers use to measure the apparent brightness (or apparent magnitude) of an object.
- With a filter that only allows light from a very specific band of wavelengths.
- Most common filters are U(\(\lambda_c=3600\)), B(\(\lambda_c=4300\)), and V(\(\lambda_c=5500\)).
- Color is the ratio of the fluxes at different wavelengths.

\[
\text{color index} = m_B - m_V = -2.5 \log_{10} \left( \frac{b_B}{b_V} \right)
\]

- By convention an (average) A0 star has U=B=V=0
Stellar Spectra

– **Chemical Composition of Stars**

**Box 19-4**

- Absorption lines present in the atmospheres of stars describe the stars chemical composition and pressure.
- Spectra of different stars shows different absorption lines. We can classify the stars based on their absorption lines.

– **The stellar classification scheme**

<table>
<thead>
<tr>
<th>O</th>
<th>B</th>
<th>A</th>
<th>F</th>
<th>G</th>
<th>K</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hottest</td>
<td>25,000K</td>
<td>Coolest</td>
<td>3000K</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Some people memorize this by saying: “Oh, Be A Fine Girl/Guy, Kiss Me”.
- Classification is based on temperature as stars of the same surface temperature show the same set of absorption lines.
- Finer classification if classes are subdivided (G1, G2 ...).
- Subclasses 0-9 also trace the temperature of a star 0 (hottest) 9 (coolest).
Stellar Classification

- Temperature determines which lines are present (too hot H is ionized - no Balmer lines, too cool H is in lowest energy state - no Balmer lines).
- Balmer lines (in absorption) are prominent as we move from B0 to A0. From A0 to F/G Balmer lines weaken.
- Molecules (TiO) can be seen in the atmospheres of M stars (3,500 degrees Kelvin).
- New classes of very cool stars (L stars) are being discovered with new surveys.
Stellar Properties

- Three fundamental stellar properties determined from observations:
  
  • **Spectral Type:** A measure of a star’s surface temperature. Determined from a star’s color and spectral lines. The spectral lines provide some information on the size/luminosity of the emitting stars.
  
  • **Luminosity (or Absolute Magnitude):** A measure of a star’s actual or intrinsic brightness. Can be determined from a star’s apparent brightness and its distance from the Earth.
  
  • **Stellar Diameter.** A star’s diameter (D) can sometimes be observed or it can be determined given a star’s surface temperature (T) and luminosity (L).
    
    - The Stefan-Boltzmann Law \((F = \sigma T^4)\): calculate a star’s luminosity given surface temperature and radius \((L = F4\pi R^2)\).
    
    - If the surface temperature is known from spectral observations, the radius of a star can be inferred from its luminosity.
Hertzsprung-Russell Diagram

• Stellar Properties are Correlated
  – Luminosity, Radius, Temperature

  Figure 19-13, 19-14
  • Luminosity is dependent on radius.
    \[ L = 4\pi R^2 \sigma T^4 \]

• Stars can be described by their luminosity and temperature (spectral type, color).
• Plotting stars on a color, luminosity diagram (Hertzsprung-Russell diagram) produces a very tight relation.
• Central band of stars is the main sequence. Contains stars undergoing “Hydrogen Burning”.
• Hot stars are more luminous than cool stars (on the main sequence).

Above the main sequence are a group of stars that are cool (red) and bright. To produce this energy they have to have large radii (Giants)

• Giants are 10-100x larger than the Sun (T=3000-6000K).
Distribution of stars

- Some giants are 1000x radius of the Sun and are called supergiants.
- Giants undergo different nuclear processes that generate the energy (Hydrogen shell burning and Helium burning - Chapters 21, 22).
- Below the main sequence are hot but have low luminosity (White Dwarfs). For their luminosities to be low they are small (size of the Earth).

Details of the Stellar Spectra

Figure 19-16

- The shapes of the absorption lines are influenced by the size of the star. Stars are further classified based on line shape. e.g. MS(V), giant (III), or supergiant (I)
  - Sun: G2 V star (i.e. G2 MS, T=5800K)
- Denser stars have broader lines (more collisions). Giants have low density atmospheres and narrow emission lines.
• **Spectroscopic Parallax**

  – **Distances from spectral properties**

  • Stellar spectra determine the type of the star (where it lies in the HR diagram).
  • From the measured colors we can derive the stars temperature and then read off its absolute (intrinsic) luminosity.
  • If we observe the apparent luminosity we have a measure of the stars distance through

  \[ L = 4\pi d^2 b \]

  • b: apparent brightness
  d: distance
  L: absolute luminosity
  • Not directly related to parallax.
Binary Stars

- **Binary stars and stellar masses**
  - **Measuring the masses of stars**
    - Binary stars are a pair of stars that orbit about the center of mass of the system.
    - Using Kepler’s Law we can measure the mass of the stars within the system.

\[
P^2 = \frac{4\pi^2}{G(M_1 + M_2)}a^3
\]

- If we measure the stars masses in solar masses (\(M_\odot\)) then this becomes

\[
M_1 + M_2 = \frac{a^3}{P^2}
\]

- \(M_1+M_2\): Sum of stellar masses (\(M_\odot\))
- \(P\): Period (years)
- \(a\): semi-major axis (AU)

- We can derive the ration of \(M_1\) and \(M_2\) from the orbit of the stars around the center of mass.
- \(M_1+M_2\) and \(M_1/M_2\) gives \(M_1\) and \(M_2\).
- Applying this relation is complicated by the tilt of the orbit of 2 stars.
– Mass Luminosity Relations

Figure 19-21

• For main sequence stars there is a tight correlation between mass and luminosity.

• This suggests that main sequence stars undergo the same physical processes in generating their energy.

\[ L = M^{3.5} \]

• L: absolute luminosity \((L_\odot)\)
  M: mass \((M_\odot)\)

• More massive stars are more luminous.

• O, B, A, F, G, K, M decreasing luminosity, mass, temperature

• For giants, super giants and white dwarfs there is no simple mass-luminosity relation.