MODERN HADRONIC RESONANCES
THEORY

by

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Hi Norbert,

I thought you might be interested:

- **THEORY POSTDOC** in **HADRONIC PHYSICS** and low energy QCD

**Physics Field(s):** nuclear physics, medium energy

**Job Description:** The University of Pittsburgh Medium Energy Physics Group invites applications for a postdoctoral research associate position beginning in Fall, 2001.

- The candidate should have an interest in **theoretical QCD in the resonance region**

The candidate will also be expected to devote a fraction of his or her time to issues relevant to the **N* PROGRAM AT JEFFERSON LAB**. The Medium Energy Group currently consists of S. DYTMAN, J. Mueller, V. Savinov, E. SWANSON, and F. Tabakin.

Norbert was here April 2002
BARYON RESONANCE EXTRACTION FROM $\pi N$ DATA USING A UNITARY MULTICHANNEL MODEL

T.P. VRANA, S.A. DYTMAN, T.-S.H. LEE
$S_{11}(1535)$ confusion

<table>
<thead>
<tr>
<th>FIT</th>
<th>$\Gamma_{\text{full}}$(MeV)</th>
<th>$b f_{\pi N}$</th>
<th>$A_{\frac{1}{2}}^p$</th>
<th>reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>VPI(96)</td>
<td>105</td>
<td>0.31</td>
<td>60 ± 15</td>
<td>$\pi N \rightarrow \pi N$, $\gamma p \rightarrow \pi p$</td>
</tr>
<tr>
<td>Drechsel(99)</td>
<td>80</td>
<td>0.40*</td>
<td>67</td>
<td>$\gamma p \rightarrow \pi p$</td>
</tr>
<tr>
<td>Krusche(97)</td>
<td>212</td>
<td>0.45*</td>
<td>120</td>
<td>$\gamma p \rightarrow \eta p$</td>
</tr>
<tr>
<td>Sauermann(96)</td>
<td>162</td>
<td>0.41</td>
<td>102 ± 20</td>
<td>$\pi N \rightarrow \pi N, \gamma p \rightarrow \pi, \eta p$</td>
</tr>
<tr>
<td>Pitt-ANL(00)</td>
<td>126</td>
<td>0.34</td>
<td>87 ± 3</td>
<td>All</td>
</tr>
<tr>
<td>Feuster(99-00)</td>
<td>151-215</td>
<td>$\sim$ 0.31</td>
<td>91-106</td>
<td>All</td>
</tr>
<tr>
<td>PDG</td>
<td>100-250</td>
<td>0.35-0.55</td>
<td>90 ± 30</td>
<td>averaging</td>
</tr>
</tbody>
</table>

* uses PDG value

thanks to Steve Dytman
the little page with the big statements

“we shall overcome” … “technical” … “food for mathematicians and philosophers” Not really! Extracting microscopic information

- Unstable states are hard to handle consistently in field theory (arrow-of-time, unitarity)

- One cannot postulate $m + i \Gamma$ without a microscopic model for the interaction and decay channels
ELECTRO PROBE of HADRONIC PROCESSES

\[ e \, h \rightarrow e' \, X \]

(both cartoon version)

- Resonances
- Polynomial Background
- Breit-Wigner

Fit of the data with BWs+Polyn.?

*What did we learn?  *What do the parameters mean?
Hamiltonian: two discrete states $a$ and $b$, one continuum $\epsilon$.

\[ H = |a\rangle m_a \langle a| + |b\rangle m_b \langle b| + \int_0^1 d\epsilon \epsilon \langle \epsilon | \langle \epsilon | \]

\[ + \int_0^1 d\epsilon g\sqrt{\epsilon(1-\epsilon)}[|a\rangle \langle \epsilon| + |b\rangle \langle \epsilon| + |\epsilon\rangle \langle a| + |\epsilon\rangle \langle b|] \]

where $|\epsilon\rangle \sim \int dk [PS]|k\rangle$. Wave function (for energy $\omega : 0 < \omega < 1$):

\[ |\omega\rangle = \alpha_a |a\rangle + \alpha_b |b\rangle + \int d\epsilon \beta(\epsilon) |\epsilon\rangle \]

\[ \Rightarrow \beta = \left( \frac{1}{\omega - \epsilon} + z(\omega) \delta(\omega - \epsilon) \right) g\sqrt{\epsilon(1-\epsilon)}(\alpha_a + \alpha_b) \]

Inserting $\beta$ back gives $(\omega - H) \cdot \alpha = 0$, hence $\det[\omega - H] = 0$ yields $z$:

\[ z(\omega) = \frac{1}{\omega(1-\omega)} \left( \left( \frac{g^2}{\omega - m_b} + \frac{g^2}{\omega - m_a} \right)^{-1} - (\omega - \frac{1}{2}) - \omega(1-\omega) \log \left| \frac{\omega}{1-\omega} \right| \right) \]
Some properties

perturbative definition

\[ \Gamma = |\langle a | H | \epsilon \rangle|^2 = g^2 \epsilon (1 - \epsilon) \]

The phase shift

\[ \delta_r = \arctan \left( \frac{-\pi}{z(\omega)} \right) \]

Scattering amplitude

\[ T = \frac{1}{z(\omega) + i\pi} \approx_{g \to 0} \frac{g^2 \omega (1 - \omega)}{(\omega - m_a)(\omega - m_b)/(2\omega - m_a - m_b) + i\pi g^2 \omega (1 - \omega)} \]

Some examples:
Real amplitude ——— Imaginary amplitude ———

Scattering energy ———

Weak coupling

Argand (2X)

Strong coupling

Argand (2X)
Real amplitude ———-
Imaginary amplitude ———-
Scattering energy ———-

Weak coupling

Argand (2X)

Strong coupling

Argand (2X)
T-Matrix / S-Matrix

\[ V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \]

nothing new

Green's Function / Propagator / Resolvent

\[ \frac{1}{E-H_0} V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \]

Eigenstates / Möller Operator

\[ \frac{1}{E-H_0} V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \frac{1}{E-H_0} V \phi_0 \]

It all boils down to evaluating:

\[ \sum_i^N \left( \frac{1}{E-H_0} V \right)^i \]
THE CORE

- Approximations at the level of the Hamiltonian (state selection)
- Maintaining unitarity and analyticity
- Restricting parameters through quantum field theory
- Renormalization (No fitting with cut-offs)
Fano in a nutshell

**THE HAMILTONIAN (Type I)**

\[ H = \sum_{i=1}^{k} |i\rangle m_i \langle i| + \int d\epsilon |\epsilon\rangle \epsilon \langle \epsilon| \]

+ \sum_{i=1}^{k} \int W_i(\epsilon) d\epsilon \left( |\epsilon\rangle e^{-i\phi_i(\epsilon)} \langle i| + |i\rangle e^{i\phi_i(\epsilon)} \langle \epsilon| \right),

**THE “EIGENSTATE” WITH ENERGY \( \omega \)**

\[ |\omega\rangle = \int d\epsilon \beta(\omega, \epsilon) |\epsilon\rangle + \sum_{i=1}^{k} \alpha_i(\omega) |i\rangle. \]
Fano in a nutshell

**THE HAMILTONIAN (Type II)**

\[ H = |1\rangle m\langle 1| + \sum_{a=1}^{k} \int d\epsilon |\epsilon, a\rangle \epsilon \langle \epsilon, a| \]

\[ + \sum_{a=1}^{k} \int W_a(\epsilon) d\epsilon \ (|\epsilon, a\rangle e^{-i\phi_a(\epsilon)} \langle 1| + |1\rangle e^{i\phi_a(\epsilon)} \langle \epsilon, a|) \]

**THE "EIGENSTATES" WITH ENERGY \( \omega \)**

\[ |\omega, b\rangle = \sum_{a=1}^{k} \int d\epsilon \beta_{a}^{(b)}(\omega, \epsilon) |\epsilon, a\rangle + \alpha^{(b)}(\omega) |1\rangle \].
Summary

\[ H_I = \begin{pmatrix} m_1 & \cdots & W_1 \\ \vdots & \ddots & \vdots \\ W_1^* & \cdots & m_k \end{pmatrix} \quad \quad H_{II} = \begin{pmatrix} m_1 & W_1 & \cdots & W_k \\ W_1^* & \vdots & \ddots & \vdots \\ W_k^* & \cdots & \epsilon_k \end{pmatrix} \]

can be solved in closed form ... (Fano)

... Many more can be turned into discrete numerical problems with exact (within numerical accuracy) solutions.
Fano Type I

where the free lunch went for dinner

$\beta(\omega, \epsilon)$ in terms of the $\alpha$’s:

$$\beta(\omega, \epsilon) = \left( \frac{1}{\omega - \epsilon} + z(\omega) \delta(\omega - \epsilon) \right) \sum_{i=1}^{k} \alpha_{i}(\omega) W_{i}(\epsilon) e^{-i\phi_{i}(\epsilon)}$$

For the consistency condition on $z(\omega)$ we define:

$$F_{ji}(\xi) = W_{i}(\xi) W_{j}(\xi) e^{i(\phi_{j}(\xi) - \phi_{i}(\xi))}$$

$$\mathcal{F}_{ji}(\eta) = \frac{1}{\pi} \int \frac{d\xi F_{ij}(\xi)}{\eta - \xi}$$

$\mathcal{F}_{ji}$ is hermitian and yields the shifted, but real, energies of the discrete states:

$$z(\omega) = (W^{\dagger}(\omega) \cdot ((\omega - \epsilon) - \pi \mathcal{F}(\omega))^{-1} \cdot W(\omega))^{-1}$$
Restricting the # of parameters

Introducing universal quantities

(form factors NR formulae renormalization scale low-energy constants cut-off
(the hadronic Lagrangian is not fundamental!)

THE QFT BROOMSTICK

renormalization scale low-energy constants cut-off

form factors NR formulae

m5 Free graphic technology
OPAL (CERN) data

τ^− → pions
rho meson peak + tail

- OPAL
- \(\pi \pi^0\)
- \(3\pi \pi^0, \pi 3\pi^0\)
- MC corr.
- perturbative QCD (massless)
- naïve parton model
Hadronic tau–lepton decay:

Some QCD corrections

= 0

VMD: cancellations

The Chiral Interpretation
Many intermediate states between $\rho$ and $\pi\pi\pi$.
THE COUPLING FUNCTIONS W:

Follow from connection between the diagonal part of the field-theoretical self-energy and the corresponding quantity in Fano theory.

\[
\rho \quad g \quad \rho
\]

\[
\pi \quad g \quad \pi
\]

approximated by:
\[
\propto (k^2)k^8 dk/\omega^2
\pi
\]

\[
\rho \quad g_2 \quad \pi \quad g_2 \quad \rho
\]

\[
\pi \quad g_2 \quad \pi \quad g_2 \quad \pi
\]
COVARIANCE

adding the backward diagrams to the real part restores covariance:

\[
\int d\epsilon f(\epsilon^2) \frac{1}{\omega - \epsilon} + \int d\epsilon f(\epsilon^2) \frac{1}{\omega - (2\omega + \epsilon)} = \int d\epsilon^2 f(\epsilon^2) \frac{1}{\omega^2 - \epsilon^2}
\]

(Only in the real parts, because threshold > 1800 MeV)
Problems with multi-loop Feynman diagrams

Picking just one: Pseudo-thresholds

which turn up at successive four-momentum integrations
(or as singularities in Feynman parameters)
CLEO data + my fit

log scale

$\log_{10}$ events/0.025GeV

energy (GeV)

$10^{-4}$ events/0.025GeV

1

0.5 1 1.5 1.5

energy (GeV)

1

500

CLEO data + my fit

norbert@washington-april-2002
CLEO data + my fit

\[ \pi \pi \text{ in the presence of } \pi \pi \pi \pi \]
$10^{-4}$ events/0.025GeV

CLEO data + my fit

$\pi \pi \pi \pi$ in the presence of $\pi \pi$

energy (GeV)
CLEO data + my fit

log scale

$10^{-4}$ events/0.025GeV vs energy (GeV)

Total $\pi\pi$ and $\pi\pi\pi\pi$
CLEO data + my fit

log scale

$10^{-4}$ events/0.025GeV

energy (GeV)

barrier term included
The underlined quantities compare with the data

$\pi^4$ suppresses $\pi^2$ decay

CLEO data + my fit

total $\pi\pi$ and $\pi\pi\pi\pi$ in the presence of $\pi\pi$ and $\pi\pi\pi\pi$

$\pi\pi$ only

$\pi\pi$ in the presence of $\pi\pi\pi\pi$

Barrier term included

$10^{-4}$ events/0.025GeV

energy (GeV)

energy (GeV)
FOUR–PION DATA SUGGESTS
A WEAKER THRESHOLD BEHAVIOR

OVERALL MAGNITUDE BELOW 1.2 GeV
WITHIN 10%  (from the inclusive data)

FROM 2–PION FIT

My four–pion prediction

Events / 0.032 (GeV

2 )

Unfolded 3

π

π

0

Events / 25 MeV (2 entries / event)

Cleo

OPAL

with omega intermediate

without omega intermediate

M (4 π) (GeV)

M (s (GeV

2 ))

Events / 0.032 (GeV

2 )

Unfolded 3π π0

Tauola 2.4

(from the inclusive data)
This is just the beginning ...... foundations of modern resonance theory

PROJECTS

• Fano Type $3^V$ (multiple discrete and continuum states)

• Three-body states$^V$, t-exchange$^V$, $N\pi\pi$ final states (sic)

• Systematize renormalization

• Coupled channel analysis, numerical code
NSTAR 2002

workshop on the

PHYSICS OF EXCITED NUCLEONS

October 9-12, 2002
University of Pittsburgh
Pittsburgh, Pennsylvania, USA

(Baryon Resonance Analysis Group meeting - October 8)

Topics

- Meson production via electromagnetic and hadronic reactions
- Baryon resonance structure in quark models
- Baryon resonances in lattice QCD
- Chiral models
- Field theory models
- Resonance parameters from coupled channels fits
- Partial wave analysis and resonance parameters
- Strangeness production
- Helicity dependence of resonances and spin structure

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